

Principles of Mathematics 12
 August 2004 Provincial Examination
ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C3	21.	A	U	1.5	2	D4
2.	A	U	1.5	2	D5	22.	B	U	1.5	3	E3
3.	D	K	1.5	2	D6	23.	B	U	1.5	3	E2
4.	C	U	1.5	2	A9	24.	D	H	1.5	3	E2
5.	D	U	1.5	2	C4	25.	D	K	1.5	3	F3
6.	C	U	1.5	2	D6	26.	C	K	1.5	3	F2
7.	B	U	1.5	2	C4, C5	27.	C	H	1.5	3	F4
8.	A	H	1.5	2	C4, C8	28.	D	H	1.5	3	F1, F2
9.	B	H	1.5	2	C6, C8	29.	C	K	1.5	4	G8
10.	D	U	1.5	2	B1	30.	C	U	1.5	4	G4
11.	A	U	1.5	2	B1	31.	B	U	1.5	4	G5
12.	D	K	1.5	2	B2	32.	C	U	1.5	4	G7
13.	A	U	1.5	2	B3	33.	D	U	1.5	4	G7
14.	B	U	1.5	2	B1	34.	A	U	1.5	4	G11
15.	A	K	1.5	2	D3	35.	B	U	1.5	4	G11
16.	C	U	1.5	2	D4	36.	A	U	1.5	4	G13
17.	D	U	1.5	2	D4	37.	C	H	1.5	4	G8
18.	C	U	1.5	2	D1	38.	B	U	1.5	4	G1
19.	A	H	1.5	2	D1	39.	B	U	1.5	4	G2
20.	B	U	1.5	2	F3; D2	40.	D	U	1.5	4	G3

Multiple Choice = 60 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F1, F5
1b.	2	U	3	3	F2, F1
2.	3	U	4	3	E3
3.	4	U	5	2	C2
4a.	5	U	2	4	G11
4b.	6	H	2	4	G12
5a.	7	U	2	4	G2
5b.	8	U	2	4	G1
6.	9	H	4	2	D6
7.	10	H	4	2	C8

Written Response = 30 marks

Multiple Choice = 60 (40 questions)

Written Response = 30 (7 questions)

EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

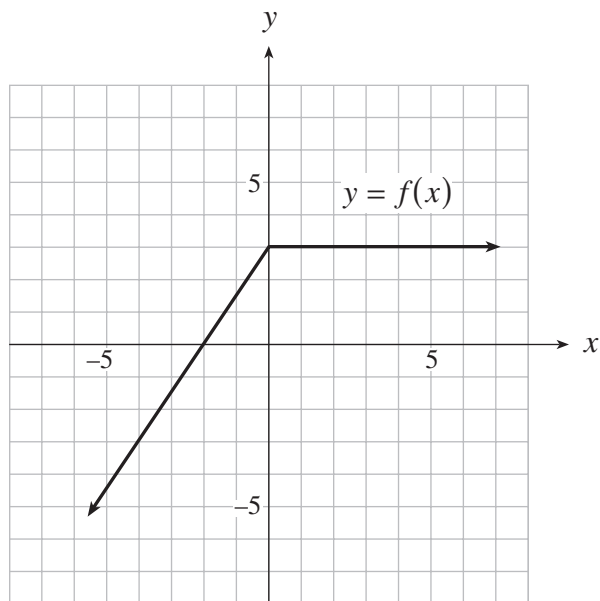
K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

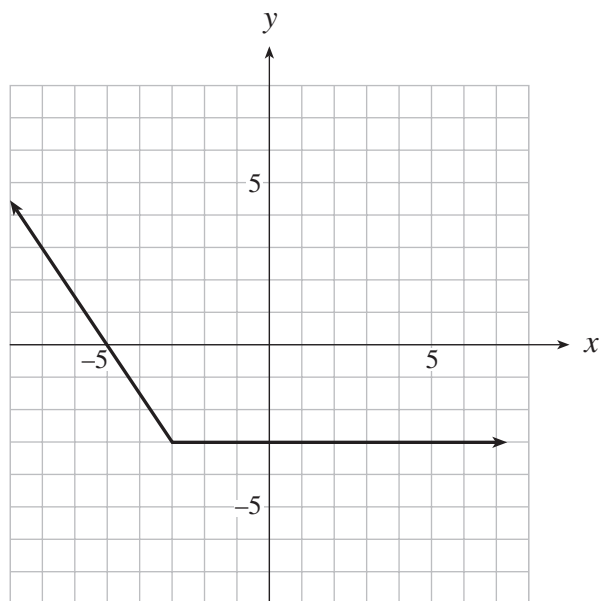
1. The function $y = f(x)$ is graphed below.



a) On the grid provided, sketch the graph of $y = -f(x + 3)$.

(2 marks)

 solution



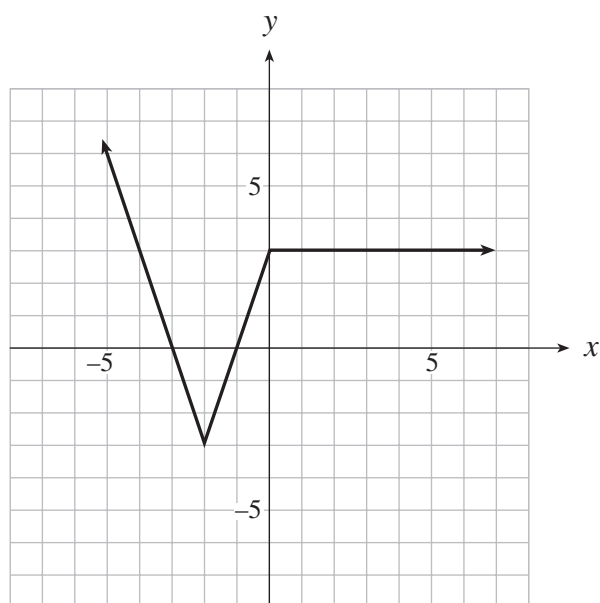
1 mark for horizontal translation

1 mark for reflection in x -axis

b) On the grid provided, sketch the graph of $y = 2|f(x)| - 3$.

(3 marks)

 solution



1 mark for absolute value

1 mark for vertical expansion

1 mark for vertical translation

2. Change $4x^2 - y^2 - 2y + 3 = 0$ to standard form.

(4 marks)

 solution

$$4x^2 - (y^2 + 2y) = -3 \quad \leftarrow \text{1 mark}$$

$$4x^2 - (y^2 + 2y + 1) = -3 - 1 \quad \leftarrow \text{1 mark}$$

$$4x^2 - (y + 1)^2 = -4 \quad \leftarrow \text{1 mark}$$

$$\left. \begin{array}{l} x^2 - \frac{(y+1)^2}{4} = -1 \\ \text{or} \\ \frac{(y+1)^2}{4} - x^2 = 1 \end{array} \right\} \quad \leftarrow \text{1 mark}$$

3. Solve algebraically: $\log(3-x) + \log(4-3x) - \log x = \log 7$

(5 marks)

solution

$$\log(3-x) + \log(4-3x) = \log 7 + \log x$$

$$\log(3-x)(4-3x) = \log 7x \quad \leftarrow 1 \text{ mark}$$

$$(3-x)(4-3x) = 7x \quad \leftarrow 1 \text{ mark}$$

$$12 - 13x + 3x^2 = 7x$$

$$3x^2 - 20x + 12 = 0 \quad \leftarrow 1 \text{ mark}$$

$$(3x-2)(x-6) = 0$$

$$x = \frac{2}{3}, 6 \quad \leftarrow 1 \text{ mark (for both solutions)}$$

↑
reject $\leftarrow 1 \text{ mark}$

$$\therefore x = \frac{2}{3}$$

alternate solution

$$\log(3-x) + \log(4-3x) - \log x = \log 7$$

$$\log \frac{(3-x)(4-3x)}{x} = \log 7 \quad \leftarrow 1 \text{ mark}$$

$$\frac{12-13x+3x^2}{x} = 7 \quad \leftarrow 1 \text{ mark}$$

$$3x^2 - 13x + 12 = 7x$$

$$3x^2 - 20x + 12 = 0 \quad \leftarrow 1 \text{ mark}$$

$$(3x-2)(x-6) = 0$$

$$x = \frac{2}{3}, x = 6 \quad \leftarrow 1 \text{ mark (for both solutions)}$$

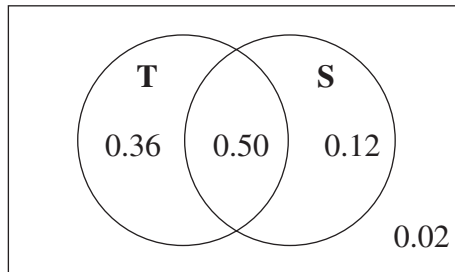
↑
reject $\leftarrow 1 \text{ mark}$

4. In one of the provinces, 86% of all homes have a television, 50% of all homes have a television and a stereo, and 2% have neither a television nor a stereo.

a) What is the probability that a randomly selected home in this province has a stereo?

(2 marks)

 **solution**



← 1 mark

$$P(S) = 0.50 + 0.12 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= 0.62 \quad \leftarrow \frac{1}{2} \text{ mark}$$

 **alternate solution**

$$P(T \text{ or } S) = P(T) + P(S) - P(T \text{ and } S) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$0.98 = 0.86 + P(S) - 0.50 \quad \leftarrow 1 \text{ mark}$$

$$0.62 = P(S) \quad \leftarrow \frac{1}{2} \text{ mark}$$

b) Given that a randomly selected home in this province has a television, what is the probability that this home does **not** have a stereo?

(2 marks)

 **solution**

$\frac{1}{2}$ mark for concept of conditional probability

$$P(\text{only T} \mid \text{have T}) = \frac{0.36}{0.86}$$

↓

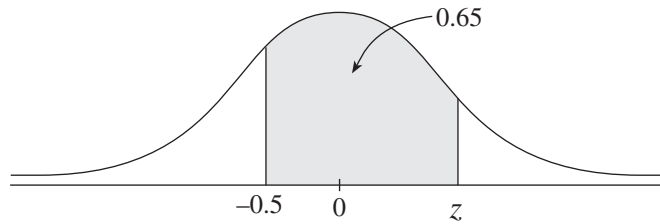
← $\frac{1}{2}$ mark

← $\frac{1}{2}$ mark

or

$$\left. \begin{array}{l} \frac{18}{43} \\ \text{or} \\ 0.42 \end{array} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

5. a) The shaded area under the standard normal curve shown is 0.65. Determine z . **(2 marks)**

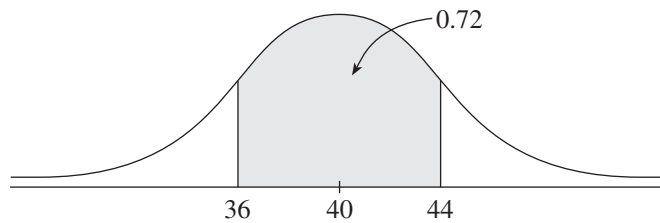


solution

$$\text{normcdf}(-1E99, -0.5) = 0.3085 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$z = \text{invNorm}(0.3085 + 0.65) = 1.73 \quad \leftarrow \mathbf{1 \text{ mark}}$$

- b) The shaded area under the normal curve shown is 0.72. If the mean for this distribution is 40, determine the standard deviation. **(2 marks)**



solution

$$\text{invNorm}(0.14) = -1.0803 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$-1.0803 = \frac{36 - 40}{\sigma} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\sigma = \frac{36 - 40}{-1.0803}$$

$$\sigma = 3.70 \quad \leftarrow \frac{1}{2} \text{ mark}$$

6. A sinusoidal curve has a minimum point at $\left(-\frac{\pi}{3}, -5\right)$ and the closest maximum point to the right is $\left(\frac{\pi}{6}, 3\right)$. Determine an equation of this curve. **(4 marks)**

 **solution**

$$\begin{array}{cccc}
 \mathbf{1\,mk} & \mathbf{1\,mk} & \mathbf{1\,mk} & \mathbf{1\,mk} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 y = 4 \cos 2\left(x - \frac{\pi}{6}\right) - 1
 \end{array}$$

or

$$y = -4 \cos 2\left(x + \frac{\pi}{3}\right) - 1$$

or

$$y = 4 \sin 2\left(x + \frac{\pi}{12}\right) - 1$$

or

$$y = -4 \sin 2\left(x - \frac{5\pi}{12}\right) - 1$$

7. Prove the identity:

(4 marks)

$$\frac{1 - \cos 2x}{\sin 2x} = \frac{1 + \tan x}{1 + \cot x}$$

 solution

LEFT SIDE	RIGHT SIDE
$\frac{1 - \cos 2x}{\sin 2x}$	$\frac{1 + \tan x}{1 + \cot x} \quad \frac{1}{2} \text{ mark}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1 - (1 - 2 \sin^2 x)}{2 \sin x \cos x}$	$= \frac{1 + \tan x}{1 + \frac{1}{\tan x}} \times \frac{\tan x}{\tan x}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{2 \sin^2 x}{2 \sin x \cos x}$	$= \frac{(1 + \tan x) \tan x}{\tan x + 1} \leftarrow \frac{1}{2} \text{ mark}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin x}{\cos x}$	$= \tan x$
	$= \frac{\sin x}{\cos x} \quad \leftarrow \frac{1}{2} \text{ mark}$

LS = RS

7. Prove the identity:

(4 marks)

$$\frac{1 - \cos 2x}{\sin 2x} = \frac{1 + \tan x}{1 + \cot x}$$

 solution

LEFT SIDE	RIGHT SIDE
$\frac{1 - \cos 2x}{\sin 2x}$	$\frac{1 + \tan x}{1 + \cot x}$
$\frac{1}{2}$ mark \rightarrow $1 - (1 - 2 \sin^2 x)$	$\frac{1}{2}$ mark \downarrow
$\frac{1}{2}$ mark \rightarrow $= \frac{2 \sin x \cos x}{2 \sin x \cos x}$	$\frac{1}{2}$ mark \downarrow
$\frac{1}{2}$ mark \rightarrow $= \frac{2 \sin^2 x}{2 \sin x \cos x}$	$= \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}} \times \frac{\sin x \cos x}{\sin x \cos x}$
$\frac{1}{2}$ mark \rightarrow $= \frac{\sin x}{\cos x}$	$= \frac{\sin x \cos x + \sin^2 x}{\sin x \cos x + \cos^2 x} \quad \leftarrow \frac{1}{2}$ mark
	$= \frac{\sin x (\cos x + \sin x)}{\cos x (\sin x + \cos x)}$
	$= \frac{\sin x}{\cos x} \quad \leftarrow \frac{1}{2}$ mark

LS = RS

END OF KEY