# Principles of Mathematics 12

June 2004 Provincial Examination

## ANSWER KEY / SCORING GUIDE

**CURRICULUM:**

| Organizers | | Sub-Organizers |
|---|---|---|
| 1. Problem Solving | A | Problem Solving and Cross Topic Problems |
| 2. Patterns and Relations | B | Geometric Sequences and Series |
| | C/D | Logarithms and Exponents |
| | C/D | Trigonometry |
| 3. Shape and Space | E | Conics |
| | F | Transformations |
| 4. Statistics and Probability | G | Combinatorics |
| | G | Probability |
| | G | Statistics |

**Part A: Multiple Choice**

| Q | K | C | S | CO | PLO | Q | K | C | S | CO | PLO |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | A | U | 1.5 | 2 | C4 | 21. | D | H | 1.5 | 2 | D1 |
| 2. | C | U | 1.5 | 2 | C3 | 22. | A | K | 1.5 | 3 | E2 |
| 3. | B | U | 1.5 | 2 | C5 | 23. | A | U | 1.5 | 3 | E3 |
| 4. | C | K | 1.5 | 2 | D6, F4 | 24. | C | H | 1.5 | 3 | E2 |
| 5. | D | U | 1.5 | 2 | C7 | 25. | A | K | 1.5 | 3 | F1 |
| 6. | D | U | 1.5 | 2 | C6 | 26. | A | U | 1.5 | 3 | F3 |
| 7. | B | U | 1.5 | 2 | D5 | 27. | D | U | 1.5 | 3 | F6 |
| 8. | A | U | 1.5 | 2 | C8 | 28. | C | H | 1.5 | 3 | F6 |
| 9. | B | H | 1.5 | 2 | D6 | 29. | D | K | 1.5 | 4 | G6 |
| 10. | A | U | 1.5 | 2 | B1 | 30. | A | U | 1.5 | 4 | G8 |
| 11. | D | U | 1.5 | 2 | B1 | 31. | C | H | 1.5 | 4 | G4 |
| 12. | B | K | 1.5 | 2 | B3 | 32. | C | U | 1.5 | 4 | G7 |
| 13. | D | U | 1.5 | 2 | B1 | 33. | D | H | 1.5 | 4 | G7 |
| 14. | D | H | 1.5 | 2 | B3 | 34. | B | U | 1.5 | 4 | G11 |
| 15. | B | K | 1.5 | 2 | D3 | 35. | B | U | 1.5 | 4 | G13 |
| 16. | C | K | 1.5 | 2 | C2 | 36. | B | U | 1.5 | 4 | G8 |
| 17. | D | U | 1.5 | 2 | C1 | 37. | B | H | 1.5 | 4 | G12 |
| 18. | C | U | 1.5 | 2 | C2 | 38. | A | U | 1.5 | 4 | G2 |
| 19. | C | U | 1.5 | 2 | C2 | 39. | C | U | 1.5 | 4 | G1 |
| 20. | A | H | 1.5 | 2 | D1 | 40. | B | H | 1.5 | 4 | G2 |

**Multiple Choice = 60 marks**

**Part B: Written Response**

| Q | B | C | S | CO | PLO |
|---|---|---|---|---|---|
| 1a. | 1 | U | 2 | 3 | F6 |
| 1b. | 2 | U | 3 | 3 | F4 |
| 2. | 3 | U | 4 | 3 | E2 |
| 3. | 4 | U | 5 | 2 | D1, D3 |
| 4. | 5 | U | 4 | 4 | G12 |
| 5a. | 6 | U | 2 | 4 | G8, G12 |
| 5b. | 7 | U | 2 | 4 | G8, G3 |
| 6. | 8 | U | 4 | 2 | D7 |
| 7. | 9 | H | 4 | 2 | C8 |

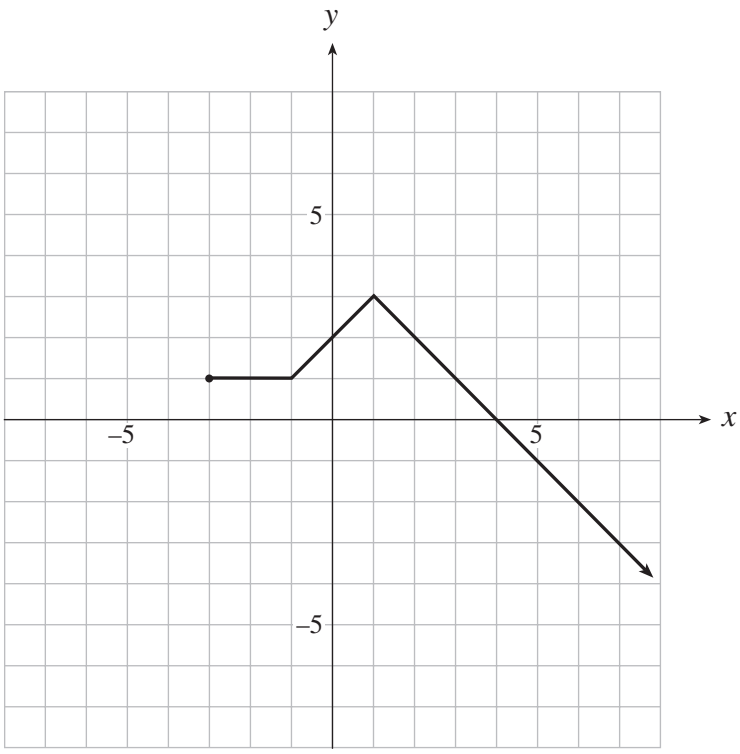
**Written Response = 30 marks**

Multiple Choice = 60 (40 questions)
Written Response = 30 (7 questions)
**EXAMINATION TOTAL = 90 marks**

**LEGEND:**

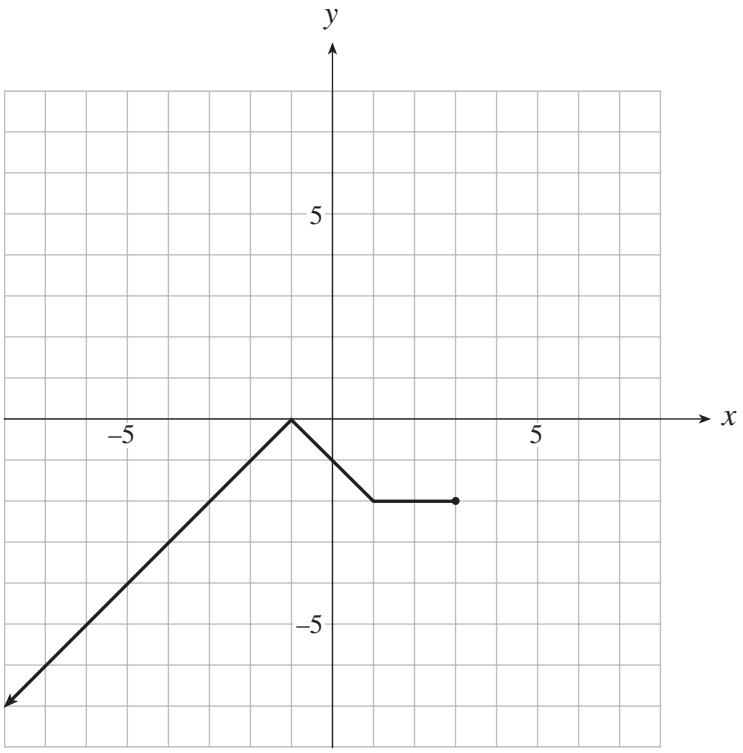
**Q** = Question Number
**B** = Score Box Number
**PLO** = Prescribed Learning Outcome
**K** = Keyed Response
**S** = Score
**C** = Cognitive Level
**CO** = Curriculum Organizer

1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = f(-x) - 3$. **(2 marks)**

**solution**

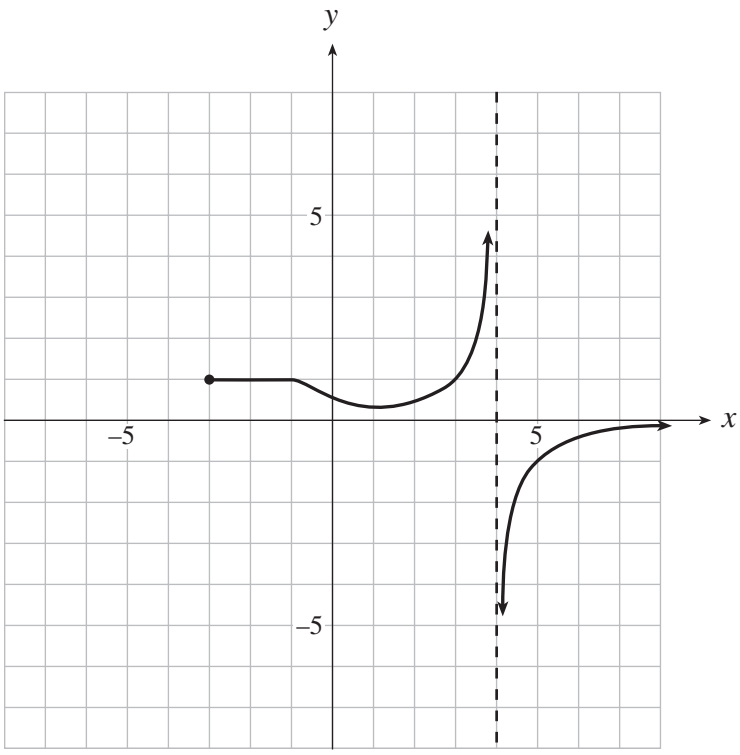



**1 mark** reflection over $y$-axis
**1 mark** translation down 3

**Note:** Deduct $\frac{1}{2}$ mark if graph does not stop on the right hand side.

b) On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(3 marks)**

## solution




$\frac{1}{2}$ **mark** for vertical asymptote or asymptotic behaviour $(x = 4)$

$\frac{1}{2}$ **mark** for horizontal asymptote on right or asymptotic behaviour $(y = 0)$

$\frac{1}{2}$ **mark** for invariant point $(5, -1)$

$\frac{1}{2}$ **mark** for invariant point $(3, 1)$

$\frac{1}{2}$ **mark** for horizontal portion $-3 \le x \le -1$

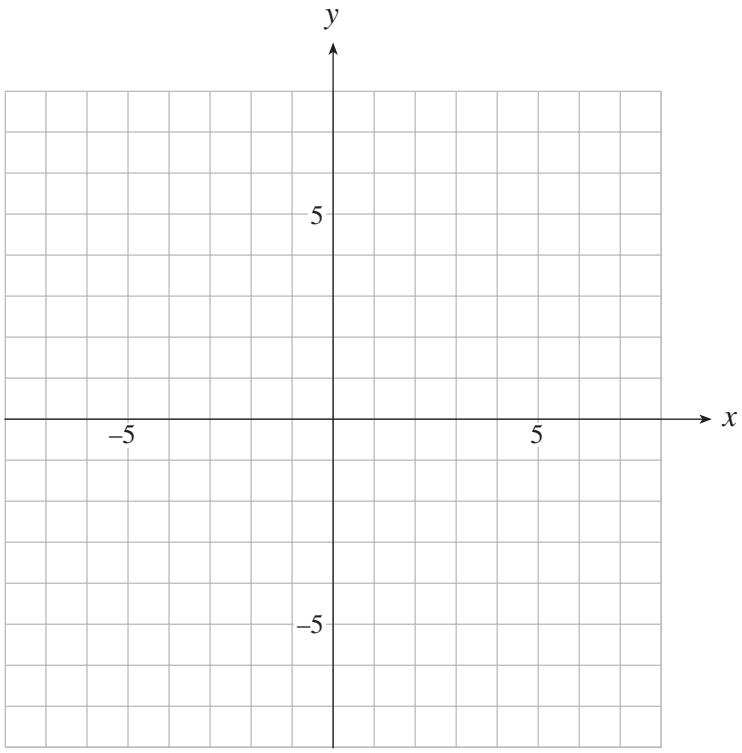
$\frac{1}{2}$ **mark** for shape from $-1 \le x \le 3$

**Note:** Deduct $\frac{1}{2}$ mark if graph does not stop at $(-3, 1)$.

**Note:** Arrowheads not necessary for full marks.

2. Determine the equation in standard form of the parabola with vertex $(5, -2)$, passing through the point $(2, 0)$, and having a horizontal axis of symmetry. **(4 marks)**

**Grid is provided for rough work only.**



**solution**

$x = a(y-k)^2 + h$ ← **1 mark**

$x = a(y+2)^2 + 5$ ← **1 $\frac{1}{2}$ marks**

$2 = a(0+2)^2 + 5$

$a = -\frac{3}{4}$ ← **1 $\frac{1}{2}$ marks**

$x = -\frac{3}{4}(y+2)^2 + 5$

**or**

$x - 5 = -\frac{3}{4}(y+2)^2$

**or**

$-\frac{4}{3}(x-5) = (y+2)^2$

} ← **1 mark**

3. The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12 000, how many weeks will it take for the population to reach 300 000 ants?

   (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) **(5 marks)**

## solution

$$300\,000 = 12\,000(3)^{\frac{t}{8}} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$\uparrow$ $\uparrow$ $\uparrow$

$\frac{1}{2}$ **mk** $\frac{1}{2}$ **mk** $\frac{1}{2}$ **mk**

$$25 = 3^{\frac{t}{8}}$$

$$\log 25 = \log 3^{\frac{t}{8}} \quad \leftarrow \textbf{1 mark}$$

$$\log 25 = \frac{t}{8}\log 3 \quad \leftarrow \textbf{1 mark}$$

**2 marks were given for showing how to solve an exponential equation using logs correctly.**

$$t = \frac{8\log 25}{\log 3} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = 23.44 \text{ weeks} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

## alternate solution

$$300\,000 = 12\,000(3)^{\frac{t}{8}} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$\uparrow$ $\uparrow$ $\uparrow$

$\frac{1}{2}$ **mk** $\frac{1}{2}$ **mk** $\frac{1}{2}$ **mk**

$$25 = 3^{\frac{t}{8}}$$

$$\log_3 25 = \frac{t}{8} \quad \leftarrow \textbf{1}\tfrac{1}{2}\textbf{ marks}$$

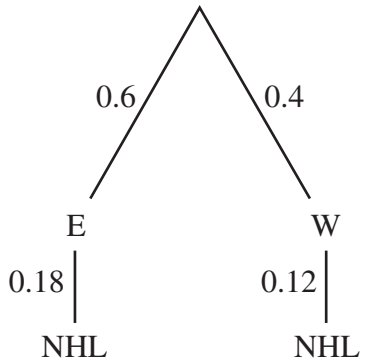
$$8\log_3 25 = t \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$8\frac{\log 25}{\log_3} = t \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$23.44 = t \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

4. In the Canadian Junior Hockey League, 60% of the players are from Eastern Canada and 40% are from Western Canada. From this league, 18% of the Eastern players and 12% of the Western players go on to play in the NHL. If a randomly chosen NHL player who came from the Canadian Junior Hockey League is selected, what is the probability that he is from Western Canada? **(4 marks)**

**solution**




$\frac{1}{2}$ **mark**

$$P(\text{W} \mid \text{NHL}) = \frac{P(\text{W and NHL})}{P(\text{NHL})} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \frac{(0.4)(0.12)}{(0.4)(0.12)+(0.6)(0.18)} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$$= 0.3077$$

$$= 0.31 \quad \leftarrow \textbf{1 mark}$$

**Note:** Deduct $\frac{1}{2}$ mark for rounding error.

5. A multiple-choice test has 48 questions. Each question has four choices, only one of which is correct. If a student answers all the questions by randomly guessing, determine the probability that the student will correctly answer between 10 and 13 questions inclusive by using the following methods.

   a) Use the binomial distribution to obtain this probability.
   (Answer accurate to at least 4 decimal places.) **(2 marks)**

## solution

$\frac{1}{2}$ **mark**

$$\underset{\frac{1}{2}\text{ mark}}{\text{binomcdf}(48, 0.25, 13)} - \underset{\frac{1}{2}\text{ mark}}{\text{binomcdf}(48, 0.25, 9)} = \underset{\frac{1}{2}\text{ mark}}{0.4938}$$

## alternate solution 1

$$\underbrace{\underset{\frac{1}{2}\text{ mark}}{\text{binompdf}(48, 0.25, 10)} + \underset{\frac{1}{2}\text{ mark}}{\text{binompdf}(48, 0.25, 11)} + \text{binompdf}(48, 0.25, 12) + \text{binompdf}(48, 0.25, 13)}_{\frac{1}{2}\text{ mark (4 terms)}} = \underset{\frac{1}{2}\text{ mark}}{0.4938}$$

## alternate solution 2

$\frac{1}{2}$ **mark**

$$\underset{\frac{1}{2}\text{ mark}}{\text{sum}}\left(\underset{\frac{1}{2}\text{ mk}}{\text{binompdf}}(48, 0.25, \{10, 11, 12, 13\})\right) \approx \underset{\frac{1}{2}\text{ mark}}{0.4938}$$

## alternate solution 3

$$\underbrace{\underset{\frac{1}{2}\text{ mark}}{{}_{48}C_{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^{38}} + \underset{\frac{1}{2}\text{ mark}}{{}_{48}C_{11}\left(\frac{1}{4}\right)^{11}\left(\frac{3}{4}\right)^{37}} + {}_{48}C_{12}\left(\frac{1}{4}\right)^{12}\left(\frac{3}{4}\right)^{36} + {}_{48}C_{13}\left(\frac{1}{4}\right)^{13}\left(\frac{3}{4}\right)^{35}}_{\frac{1}{2}\text{ mark (4 terms)}} = \underset{\frac{1}{2}\text{ mark}}{0.4938}$$

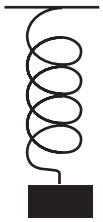
b) Use the normal approximation to the binomial distribution to obtain an estimate of this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

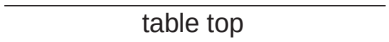
## solution

$$\left.\begin{aligned} \mu &= np = 48(0.25) = 12 \\ \sigma &= \sqrt{npq} = \sqrt{48(.25)(.75)} = 3 \end{aligned}\right\} \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\underbrace{\text{normalcdf}\,(9.5,\ 13.5,\ 12,\ 3)}_{\substack{\uparrow \\ \frac{1}{2}\textbf{ mark}}\qquad\qquad\underbrace{\qquad\qquad}_{\frac{1}{2}\textbf{ mark continuity correction}}} = 0.4891 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

6. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height $h$, in cm, above the table top, is approximated by a sinusoidal function of the elapsed time $t$, in seconds, for a short period of time.






Determine an equation for a sinusoidal function that gives $h$ as a function of $t$. **(4 marks)**

## solution

**1 mk** **1 mk**

$$h = -30\cos\frac{2\pi}{1.6}t + 50$$

**1 mark**

**1 mark for proper phase shift expression** $(t = t - 0)$

**OR**

$$h = 30\sin\frac{2\pi}{1.6}(t - 0.4) + 50$$

**OR**

$$h = 30\cos\frac{2\pi}{1.6}(t - 0.8) + 50$$

**OR**

$$h = -30\sin\frac{2\pi}{1.6}(t - 1.2) + 50$$

**OR**

$$h = -30\sin\frac{2\pi}{1.6}(t + 0.4) + 50$$

7. Prove the identity: **(4 marks)**

$$\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$$

**solution**

| LEFT SIDE | RIGHT SIDE |
|---|---|
| **both for $\frac{1}{2}$ mark** | $\sec\theta$ |
| $\frac{1}{\sin\theta}\cdot 2\sin\theta\cos\theta - \frac{1}{\cos\theta}\left(2\cos^2\theta - 1\right)$ ($\frac{1}{2}$ mk, $\frac{1}{2}$ mark) | |
| **1$\frac{1}{2}$ marks** $\rightarrow$ $2\cos\theta - 2\cos\theta + \frac{1}{\cos\theta}$ | |
| **1 mark** $\rightarrow$ $\frac{1}{\cos\theta}$ | |
| $\sec\theta$ | |
| LS = RS | |

**Note:** Deduct $\frac{1}{2}$ mark if LS and RS are not identical.

7. Prove the identity: **(4 marks)**

$$\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$$

## alternate solution 1

| LEFT SIDE | RIGHT SIDE |
|---|---|
| | $\sec\theta$ |

**both for $\frac{1}{2}$ mark**

**$\frac{1}{2}$ mark** **$\frac{1}{2}$ mark**

$$\frac{1}{\sin\theta}\cdot 2\sin\theta\cos\theta - \frac{1}{\cos\theta}\left(\cos^2\theta - \sin^2\theta\right)$$

**$1\frac{1}{2}$ marks** $\rightarrow$ $2\cos\theta - \cos\theta + \dfrac{\sin^2\theta}{\cos\theta}$

$$\cos\theta + \frac{\sin^2\theta}{\cos\theta}$$

**$\frac{1}{2}$ mark** $\rightarrow$ $\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

**$\frac{1}{2}$ mark** $\rightarrow$ $\dfrac{1}{\cos\theta}$

$$\sec\theta$$

LS = RS

**Note:** Deduct $\frac{1}{2}$ mark if LS and RS are not identical.

7. Prove the identity: **(4 marks)**

$$\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$$

## alternate solution 2

| LEFT SIDE | RIGHT SIDE |
|---|---|
| | $\sec\theta$ |
| **both for $\frac{1}{2}$ mark**; $\frac{1}{2}$ **mark**, $\frac{1}{2}$ **mark**: $\frac{1}{\sin\theta}\cdot 2\sin\theta\cos\theta - \frac{1}{\cos\theta}\left(1-2\sin^2\theta\right)$ | |
| **$1\frac{1}{2}$ marks** $\rightarrow$ $2\cos\theta - \frac{1}{\cos\theta} + \frac{2\sin^2\theta}{\cos\theta}$ | |
| **$\frac{1}{2}$ mark** $\rightarrow$ $\frac{2\cos^2\theta - 1 + 2\sin^2\theta}{\cos\theta}$ | |
| $\frac{2\left(\cos^2\theta + \sin^2\theta\right) - 1}{\cos\theta}$ | |
| $\frac{2-1}{\cos\theta}$ | |
| **$\frac{1}{2}$ mark** $\rightarrow$ $\frac{1}{\cos\theta}$ | |
| $\sec\theta$ | |
| LS = RS | |

**Note:** Deduct $\frac{1}{2}$ mark if LS and RS are not identical.

7. Prove the identity: **(4 marks)**

$$\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$$

## alternate solution 3

| LEFT SIDE | | RIGHT SIDE |
|---|---|---|
| $= \dfrac{\sin 20}{\sin\theta} - \dfrac{\cos 20}{\cos\theta}$ | $\leftarrow \frac{1}{2}$ **mark** | $\sec\theta$ |
| $= \dfrac{\sin 20\cos\theta - \sin\theta\cos 20}{\sin\theta\cos\theta}$ | $\leftarrow \frac{1}{2}$ **mark** | |
| $= \dfrac{\sin(20-\theta)}{\sin\theta\cos\theta}$ | $\leftarrow$ **1** $\frac{1}{2}$ **marks** | |
| $= \dfrac{\sin\theta}{\sin\theta\cos\theta}$ | $\leftarrow$ **1 mark** | |
| $= \dfrac{1}{\cos\theta}$ | $\leftarrow \frac{1}{2}$ **mark** | |
| $= \sec\theta$ | | |

LS = RS

**Note:** Deduct $\frac{1}{2}$ mark if LS and RS are not identical.

**END OF KEY**