

**Principles of Mathematics 12**  
 April 2004 Provincial Examination  
**ANSWER KEY / SCORING GUIDE**

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**CURRICULUM:**

<b>Organizers</b>									
1. Problem Solving	A	Problem Solving and Cross Topic Problems							
2. Patterns and Relations	B	Geometric Sequences and Series							
	C/D	Logarithms and Exponents							
	C/D	Trigonometry							
3. Shape and Space	E	Conics							
	F	Transformations							
4. Statistics and Probability	G	Combinatorics							
	G	Probability							
	G	Statistics							

**Part A: Multiple Choice**

<b>Q</b>	<b>K</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>	<b>Q</b>	<b>K</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
1.	A	U	1.5	2	C3	21.	D	H	1.5	2	D3
2.	D	K	1.5	2	D5	22.	C	K	1.5	3	E1
3.	B	U	1.5	2	C4	23.	B	U	1.5	3	E2
4.	A	U	1.5	2	D6	24.	D	H	1.5	3	E2
5.	D	U	1.5	2	C3	25.	B	U	1.5	3	F5
6.	B	U	1.5	2	C5	26.	A	U	1.5	3	F2
7.	A	U	1.5	2	C7, C5	27.	A	U	1.5	3	F6
8.	B	U	1.5	2	C8	28.	C	H	1.5	3	F1, F3; A2
9.	A	H	1.5	2	C4; B1; A2	29.	B	U	1.5	4	G7, G8
10.	C	K	1.5	2	B1	30.	B	U	1.5	4	G6
11.	C	U	1.5	2	B1	31.	D	U	1.5	4	G8
12.	C	U	1.5	2	B1	32.	C	U	1.5	4	G11
13.	C	U	1.5	2	B1, B2	33.	C	U	1.5	4	G12
14.	B	H	1.5	2	B1; C2; A2	34.	B	H	1.5	4	G11
15.	D	K	1.5	2	D2	35.	A	U	1.5	4	G3
16.	B	U	1.5	2	C1	36.	A	H	1.5	4	G2
17.	D	U	1.5	2	D3	37.	C	U	1.5	4	G1
18.	D	U	1.5	2	D1	38.	C	U	1.5	4	G2
19.	D	U	1.5	2	D4	39.	A	K	1.5	4	G1
20.	C	U	1.5	2	C2	40.	B	H	1.5	4	G2

**Multiple Choice = 60 marks**

## Part B: Written Response

<b>Q</b>	<b>B</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
1.	1	U	5	2	C2
2.	2	U	4	2	E3; F1; A2
3.	3	U	5	3	F3
4a.	4	U	2	2	D7
4b.	5	U	2	2	D7
5.	6	U	4	4	G6
6a.	7	H	3	4	G12
6b.	8	H	1	4	G12
7.	9	H	4	2	C7

**Written Response = 30 marks**

Multiple Choice = 60 (40 questions)

Written Response = 30 (7 questions)

**EXAMINATION TOTAL = 90 marks**

### **LEGEND:**

**Q** = Question Number

**B** = Score Box Number

**PLO** = Prescribed Learning Outcome

**K** = Keyed Response

**S** = Score

**C** = Cognitive Level

**CO** = Curriculum Organizer

1. Solve algebraically:  $2 \log_4 x - \log_4(x + 3) = 1$

(5 marks)

 **solution**

$$\log_4 x^2 - \log_4(x + 3) = 1 \quad \leftarrow \text{1 mark}$$

$$\log_4 \frac{x^2}{x + 3} = 1 \quad \leftarrow \text{1 mark}$$

$$4 = \frac{x^2}{x + 3} \quad \leftarrow \text{1 mark}$$

$$4x + 12 = x^2$$

$$0 = x^2 - 4x - 12$$

$$0 = (x - 6)(x + 2)$$

**1 mark**  $\rightarrow$   $x = 6, \quad x = -2$   
 $\downarrow$   
reject  $\leftarrow$  **1 mark**

$$\therefore x = 6$$

2. The circle with equation  $x^2 + 6x + y^2 + 2y = 0$  is translated 2 units to the right to form a new circle. Determine the equation of the new circle and change to standard form. **(4 marks)**

### solution

$$x^2 + 6x + y^2 + 2y = 0$$

$$\underbrace{(x-2)^2}_{\uparrow \frac{1}{2} \text{ mark}} + \underbrace{6(x-2)}_{\uparrow \frac{1}{2} \text{ mark}} + y^2 + 2y = 0$$

$$(x^2 - 4x + 4) + (6x - 12) + y^2 + 2y = 0 \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 + 2x + y^2 + 2y - 8 = 0 \leftarrow \frac{1}{2} \text{ mark}$$

$$\underbrace{(x^2 + 2x + 1)}_{\uparrow \frac{1}{2} \text{ mark}} + \underbrace{(y^2 + 2y + 1)}_{\uparrow \frac{1}{2} \text{ mark}} = \underbrace{8 + 1 + 1}$$

$$\underbrace{(x+1)^2}_{\uparrow \frac{1}{2} \text{ mark}} + \underbrace{(y+1)^2}_{\uparrow \frac{1}{2} \text{ mark}} = 10$$

### alternate solution

$$x^2 + 6x + y^2 + 2y = 0$$

$$x^2 + 6x + 9 + y^2 + 2y + 1 = 9 + 1 \leftarrow \frac{1}{2} \text{ mark}$$

$$\uparrow \quad \quad \uparrow$$

$$\frac{1}{2} \text{ mark} \quad \quad \frac{1}{2} \text{ mark}$$

$$\underbrace{(x+3)^2}_{\uparrow \frac{1}{2} \text{ mark}} + \underbrace{(y+1)^2}_{\uparrow \frac{1}{2} \text{ mark}} = \underbrace{10}_{\uparrow \frac{1}{2} \text{ mark}}$$

translate 2 units  
right  $\Rightarrow$   $\underbrace{(x+1)^2}_{\uparrow \frac{1}{2} \text{ mark}} + (y+1)^2 = 10$   
**1 mark**

3. Given the function  $f(x) = \frac{3x}{x+1}$ , determine the equation of the inverse function  $f^{-1}(x)$ .

(Note: Write the inverse function in the form  $f^{-1}(x) = \text{“ } \quad \text{”}$ .)

(5 marks)

### solution

$$f(x) = \frac{3x}{x+1}$$

$$y = \frac{3x}{x+1}$$

$$x = \frac{3y}{y+1} \quad \leftarrow \text{1 mark}$$

$$x(y+1) \text{ or } xy + x = 3y \quad \leftarrow \text{1 mark}$$

$$x = 3y - xy \quad \leftarrow \text{1 mark}$$

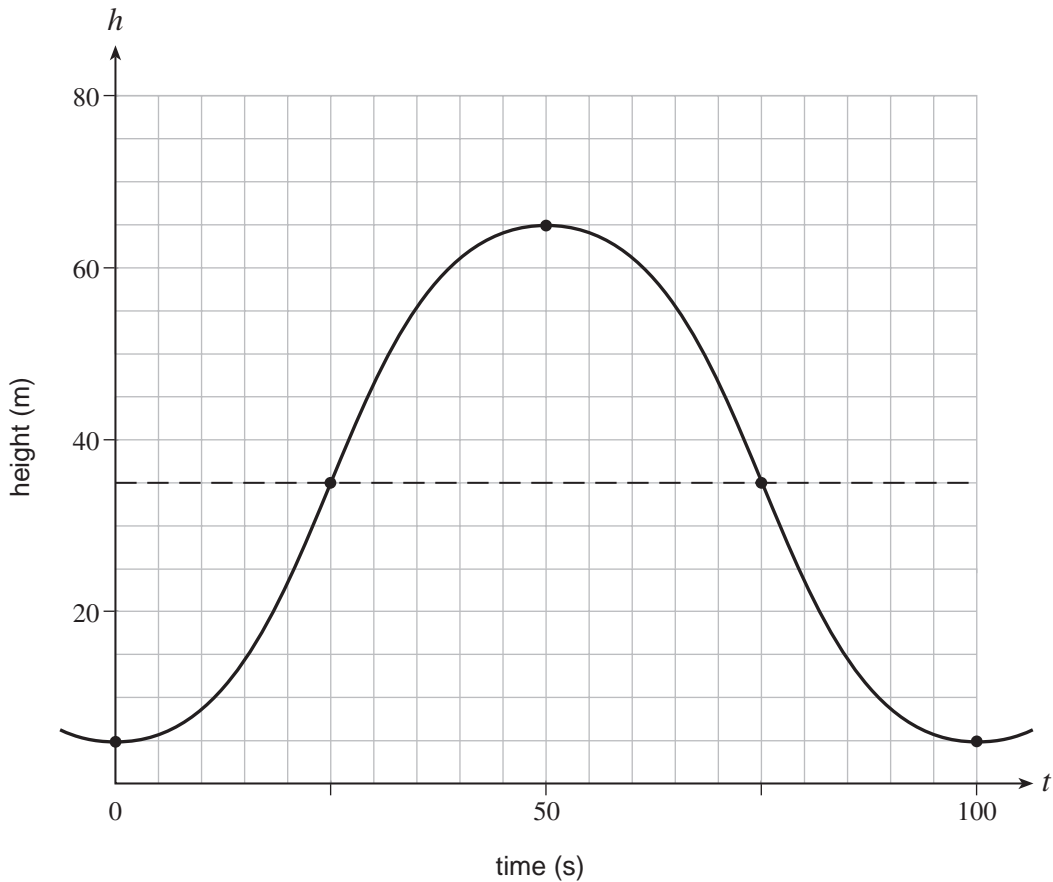
$$x = (3-x)y \quad \leftarrow \text{1 mark}$$

$$\left. \begin{array}{l} \frac{x}{3-x} = y \\ \text{or} \\ f^{-1}(x) = \frac{x}{3-x} \\ \text{or} \\ f^{-1}(x) = \frac{-x}{x-3} \end{array} \right\} \leftarrow \text{1 mark}$$

4. A Ferris wheel with a radius of 30 m rotates once every 100 s. At time  $t = 0$  s, passengers get on at the lowest point of its rotation which is 5 m above the ground.

- a) Using the grid below, graph how the height  $h$  of a passenger varies with respect to the elapsed time  $t$  during at least one rotation of the Ferris wheel. Clearly show at least 5 points on your graph and indicate the scale on the vertical axis. **(2 marks)**

 solution



**1 mark** for general shape  
**1 mark** for accuracy of 5 points

b) Determine a sine function that gives the passenger's height  $h$  metres, above the ground as a function of time  $t$  seconds. **(2 marks)**

 **solution**

$$h = 30 \sin \frac{2\pi}{100}(t - 25) + 35$$

↑    ↑    ↑    ↑  
 $\frac{1}{2}$  mk    $\frac{1}{2}$  mk    $\frac{1}{2}$  mk    $\frac{1}{2}$  mk

 **alternate solution**

$$h = -30 \sin \frac{2\pi}{100}(t - 75) + 35$$

$$h = -30 \sin \frac{2\pi}{100}(t + 25) + 35$$

**Note: 1 mark** for correct cosine function

5. Solve algebraically using factorial notation:  ${}_n P_2 = 90$

(4 marks)

 solution

$${}_n P_2 = 90$$

$$\frac{n!}{(n-2)!} = 90 \quad \leftarrow \text{1 mark}$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 90$$

$$n^2 - n = 90 \quad \leftarrow \text{1 mark}$$

$$n^2 - n - 90 = 0$$

$$(n-10)(n+9) = 0$$

$$\text{1 mark} \rightarrow n = 10 \quad n = -9$$

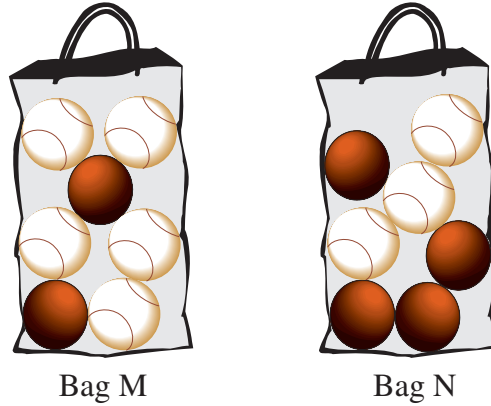
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reject  $\leftarrow \text{1 mark}$

$$\therefore n = 10$$



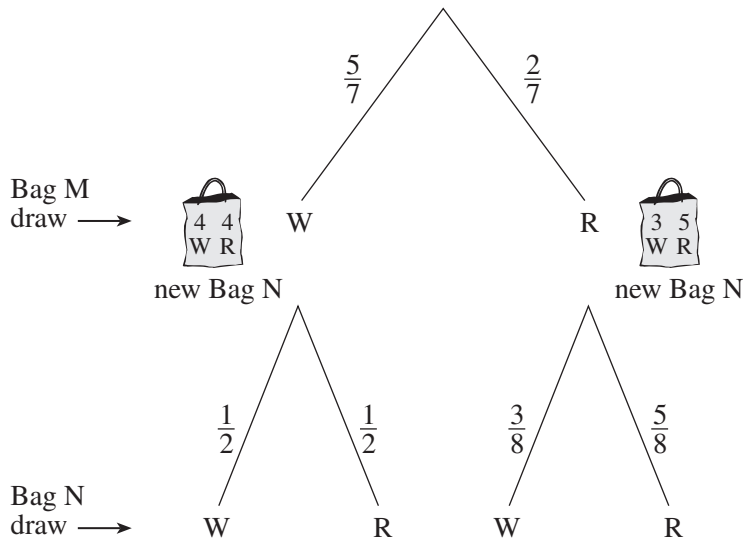
6. Bag M contains 5 white balls and 2 red balls. Bag N contains 3 white balls and 4 red balls.



a) A ball is randomly selected from Bag M and placed in Bag N. A ball is then randomly selected from Bag N. What is the probability that the ball selected from Bag N is white?

**(3 marks)**

**solution**



$$P(W_{\text{bag N}}) = \left(\frac{5}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{7}\right)\left(\frac{3}{8}\right) = \frac{13}{28} \text{ or } 0.46$$

↑  
**3 marks**

↑  
 **$\frac{1}{2}$  mark  
deducted if wrong**

b) If a white ball is selected from Bag N, what is the probability that a red ball was transferred from Bag M to Bag N? **(1 mark)**

 **solution**

$$\begin{aligned} P(R_{\text{bag M}} | W_{\text{bag N}}) &= \frac{P(R_{\text{bag M}} \text{ and } W_{\text{bag N}})}{P(W_{\text{bag N}})} \\ &= \frac{\frac{2}{7} \times \frac{3}{8}}{\left(\frac{5}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{7}\right)\left(\frac{3}{8}\right)} && \leftarrow \frac{1}{2} \text{ mark} \\ &= \frac{3}{13} \text{ or } 0.23 && \leftarrow \frac{1}{2} \text{ mark } \underline{\text{deducted}} \text{ if answer not } 0 \leq p \leq 1 \end{aligned}$$

7. Prove the identity:

(4 marks)

$$\tan \theta \cos 2\theta + \tan \theta = \sin 2\theta$$

 solution

LEFT SIDE	RIGHT SIDE
<b>1 mark</b> $\rightarrow = \tan \theta (\cos 2\theta + 1)$	$\sin 2\theta$
$\frac{1}{2}$ <b>mark</b> $\downarrow$ <b>1 mark</b> $\downarrow$ $= \frac{\sin \theta}{\cos \theta} (2 \cos^2 \theta - 1 + 1)$	
$\frac{1}{2}$ <b>mark</b> $\rightarrow = \frac{\sin \theta}{\cos \theta} \cdot 2 \cos^2 \theta$	
$\frac{1}{2}$ <b>mark</b> $\rightarrow = 2 \sin \theta \cos \theta$	
$\frac{1}{2}$ <b>mark</b> $\rightarrow = \sin 2\theta$	
LS = RS	

7. Prove the identity:

(4 marks)

$$\tan \theta \cos 2\theta + \tan \theta = \sin 2\theta$$

 alternate solution 1

LEFT SIDE	RIGHT SIDE
$= \frac{\sin \theta}{\cos \theta} \left( \overset{\frac{1}{2} \text{ mark}}{\downarrow} \cos^2 \theta - \overset{\frac{1}{2} \text{ mark}}{\downarrow} \sin^2 \theta \right) + \frac{\sin \theta}{\cos \theta}$	$\sin 2\theta$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin \theta}{\cos \theta} (\cos^2 \theta - \sin^2 \theta + 1)$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin \theta}{\cos \theta} (\cos^2 \theta + \cos^2 \theta)$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin \theta}{\cos \theta} (2 \cos^2 \theta)$	
$\frac{1}{2} \text{ mark} \rightarrow = 2 \sin \theta \cos \theta$	
$\frac{1}{2} \text{ mark} \rightarrow = \sin 2\theta$	
LS = RS	

7. Prove the identity:

(4 marks)

$$\tan \theta \cos 2\theta + \tan \theta = \sin 2\theta$$

 alternate solution 2

LEFT SIDE	RIGHT SIDE
$\begin{array}{c} \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \\ \downarrow \quad \downarrow \\ = \frac{\sin \theta}{\cos \theta} (1 - 2 \sin^2 \theta) + \frac{\sin \theta}{\cos \theta} \end{array}$	$\sin 2\theta$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin \theta}{\cos \theta} (1 - 2 \sin^2 \theta + 1)$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin \theta}{\cos \theta} (2 - 2 \sin^2 \theta)$	
$= \frac{2 \sin \theta}{\cos \theta} (1 - \sin^2 \theta)$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{2 \sin \theta}{\cos \theta} (\cos^2 \theta)$	
$\frac{1}{2} \text{ mark} \rightarrow = 2 \sin \theta \cos \theta$	
$\frac{1}{2} \text{ mark} \rightarrow = \sin 2\theta$	
LS = RS	

**END OF KEY**