

Principles of Mathematics 12
 January 2004 Provincial Examination
ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1.5	2	C3	21.	A	H	1.5	2	D3
2.	D	U	1.5	2	D6	22.	C	K	1.5	3	E2
3.	A	U	1.5	2	C5	23.	C	U	1.5	3	E3
4.	A	U	1.5	2	C4	24.	A	H	1.5	3	E2
5.	D	U	1.5	2	D5	25.	A	K	1.5	3	F1
6.	C	U	1.5	2	C6	26.	A	U	1.5	3	F3
7.	C	H	1.5	2	D6	27.	B	H	1.5	3	F2, E2
8.	B	H	1.5	2	D7	28.	C	H	1.5	3	F4, F2
9.	C	U	1.5	2	C8	29.	D	U	1.5	4	G5
10.	C	K	1.5	2	B1	30.	B	U	1.5	4	G6
11.	B	U	1.5	2	B1	31.	C	U	1.5	4	G9
12.	A	U	1.5	2	B1	32.	C	U	1.5	4	G13
13.	D	U	1.5	2	B1	33.	C	U	1.5	4	G9
14.	D	H	1.5	2	B1, G8	34.	B	H	1.5	4	G11
15.	C	U	1.5	2	C2	35.	C	K	1.5	4	G2
16.	A	K	1.5	2	D3, F3	36.	B	U	1.5	4	G2
17.	B	U	1.5	2	D4	37.	B	U	1.5	4	G2
18.	D	U	1.5	2	D2, F3	38.	D	K	1.5	4	G1, G8
19.	D	U	1.5	2	C2	39.	C	H	1.5	4	G1
20.	B	H	1.5	2	D1	40.	D	H	1.5	4	G3

Multiple Choice = 60 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	4	G7
1b.	2	U	2	4	G7
2.	3	U	5	2	C2
3a.	4	U	2	4	G8
3b.	5	U	2	4	G8
4.	6	U	4	3	E2
5a.	7	U	3	3	F6
5b.	8	U	2	3	F2, F5
6.	9	U	4	2	C5
7.	10	H	4	2	C8

Written Response = 30 marks

Multiple Choice = 60 (40 questions)

Written Response = 30 (7 questions)

EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

1. a) A theatre company of 13 actors consists of 8 men and 5 women. How many different ways are there to choose from the theatre company a group of 7 with exactly 3 men? **(2 marks)**

 **solution**

$$\begin{array}{c} \frac{1}{2} \text{ mark (multiplication sign)} \\ \frac{1}{2} \text{ mark} \swarrow \quad \downarrow \quad \searrow \quad \frac{1}{2} \text{ mark} \\ ({}_8C_3)({}_5C_4) = 280 \\ \quad \quad \quad \uparrow \\ \quad \quad \quad \frac{1}{2} \text{ mark} \end{array}$$

- b) A theatre company of 13 actors consists of 8 men and 5 women. How many different ways are there to choose from the theatre company a group of 6 with at least 4 women? **(2 marks)**

 **solution**

$$\begin{array}{c} \frac{1}{2} \text{ mark (addition sign)} \\ \downarrow \\ \underbrace{({}_8C_2)({}_5C_4)}_{\frac{1}{2} \text{ mark}} + \underbrace{({}_8C_1)({}_5C_5)}_{\frac{1}{2} \text{ mark}} = 148 \\ \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\ \quad \quad \quad \frac{1}{2} \text{ mark} \quad \quad \quad \frac{1}{2} \text{ mark} \quad \quad \quad \frac{1}{2} \text{ mark} \end{array}$$

2. Solve algebraically using logarithms: $2^x = 5^{x+1}$
(Answer accurate to at least 2 decimal places.)

(5 marks)

solution

$$2^x = 5^{x+1}$$

$$\log 2^x = \log 5^{x+1} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x \log 2 = (x+1) \log 5 \quad \leftarrow 1 \text{ mark}$$

$$x \log 2 = x \log 5 + \log 5 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x \log 2 - x \log 5 = \log 5 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x(\log 2 - \log 5) = \log 5 \quad \leftarrow 1 \text{ mark}$$

$$x = \frac{\log 5}{\log 2 - \log 5} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = -1.76 \quad \leftarrow 1 \text{ mark}$$

alternate solution

$$2^x = 5^{x+1}$$

$$\log_5 2^x = x+1 \quad \leftarrow 1 \text{ mark}$$

$$x \log_5 2 = x+1 \quad \leftarrow 1 \text{ mark}$$

$$x \log_5 2 - x = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(\log_5 2 - 1)x = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = \frac{1}{\log_5 2 - 1} \quad \leftarrow 1 \text{ mark}$$

$$x = -1.76 \quad \leftarrow 1 \text{ mark}$$

3. The probability of Bruce sinking a one-metre putt is 85%.

a) Assuming independence, what is the probability that Bruce sinks exactly 10 out of 12 one-metre putts? (Answer accurate to at least 4 decimal places.)

(2 marks)

 **solution**

$$\underbrace{\text{binompdf}}_{\uparrow \text{ 1 mark}} \left(\underbrace{12, 0.85, 10}_{\uparrow \frac{1}{2} \text{ mark}} \right) = 0.2924$$

\uparrow
 $\frac{1}{2}$ mark

OR

$$\underbrace{{}_{12}C_{10}}_{\uparrow \text{ 1 mark}} \underbrace{(0.85)^{10}(0.15)^2}_{\uparrow \frac{1}{2} \text{ mark}} = 0.2924$$

\uparrow
 $\frac{1}{2}$ mark

b) Assuming independence, what is the probability that Bruce sinks at least 10 out of 12 one-metre putts? (Answer accurate to at least 4 decimal places.)

(2 marks)

 solution

$$\begin{array}{c} \frac{1}{2} \text{ mark} \\ \downarrow \\ \overbrace{1 - \text{binomcdf}(12, 0.85, 9)} = 0.7358 \\ \underbrace{\hspace{1.5cm}} \quad \underbrace{\hspace{1.5cm}} \quad \uparrow \\ \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \end{array}$$

OR

$$\begin{array}{c} \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \\ \downarrow \quad \downarrow \\ \overbrace{\text{binompdf}(12, 0.85, 10)} = 0.292358 \\ + \\ \text{binompdf}(12, 0.85, 11) = 0.3012178 \\ + \\ \text{binompdf}(12, 0.85, 12) = 0.1422418 \end{array} \left. \vphantom{\begin{array}{c} \frac{1}{2} \text{ mark} \\ \downarrow \\ \text{binompdf}(12, 0.85, 10) \end{array}} \right\} \leftarrow \frac{1}{2} \text{ mark for recognizing 10, 11, 12}$$

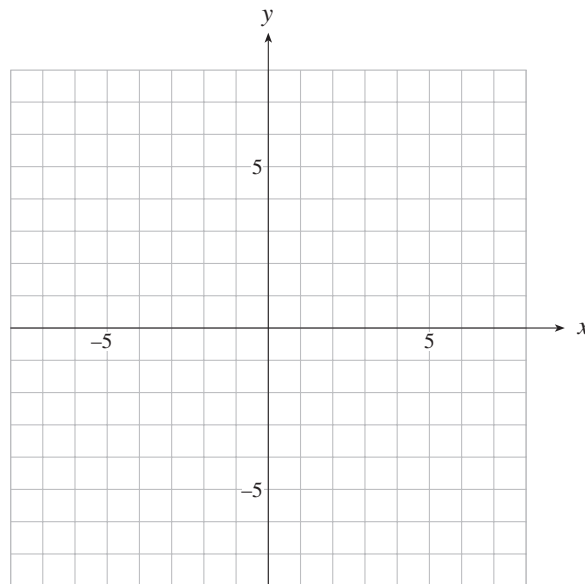
$$= 0.7358 \quad \leftarrow \frac{1}{2} \text{ mark}$$

OR

$$\begin{array}{c} \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \\ \downarrow \quad \downarrow \\ \overbrace{{}_{12}C_{10}(0.85)^{10}(0.15)^2} + \overbrace{{}_{12}C_{11}(0.85)^{11}(0.15)^1} + \overbrace{{}_{12}C_{12}(0.85)^{12}(0.15)^0} \\ = 0.29235849 + 0.301217839 + 0.142241757 \\ \underbrace{\hspace{10cm}} \\ \uparrow \\ \frac{1}{2} \text{ mark} \\ = 0.7358 \quad \leftarrow \frac{1}{2} \text{ mark} \end{array}$$

4. An ellipse has vertices $(3, -2)$ and $(3, 6)$. If the ellipse passes through the point $(0, 0)$, determine the equation of the ellipse in standard form. **(4 marks)**

Grid is provided for rough work only.



solution

1 mark centre

$$\frac{(x-3)^2}{b^2} + \frac{(y-2)^2}{16} = 1 \quad \leftarrow \frac{1}{2} \text{ mark for ellipse form}$$

1 mark for 16

$\frac{1}{2}$ mark for →
substitution
of $(0, 0)$

$$\frac{(0-3)^2}{b^2} + \frac{(0-2)^2}{16} = 1$$

$$\frac{9}{b^2} + \frac{4}{16} = 1$$

$$\frac{9}{b^2} = \frac{12}{16}$$

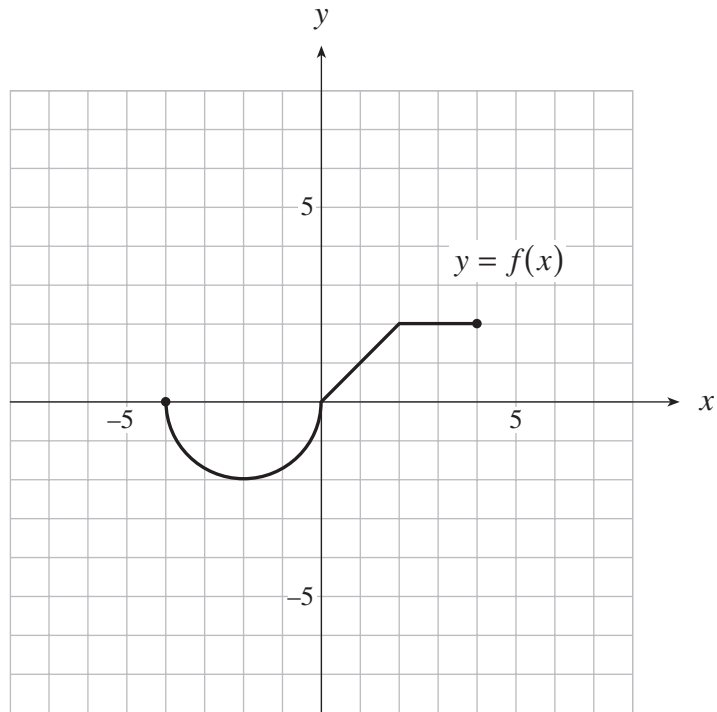
$$12 = b^2$$

$\left. \begin{array}{l} \frac{9}{b^2} + \frac{4}{16} = 1 \\ \frac{9}{b^2} = \frac{12}{16} \\ 12 = b^2 \end{array} \right\} \leftarrow \frac{1}{2} \text{ mark}$

$$\therefore \frac{(x-3)^2}{12} + \frac{(y-2)^2}{16} = 1$$

\uparrow
 $\frac{1}{2}$ mark

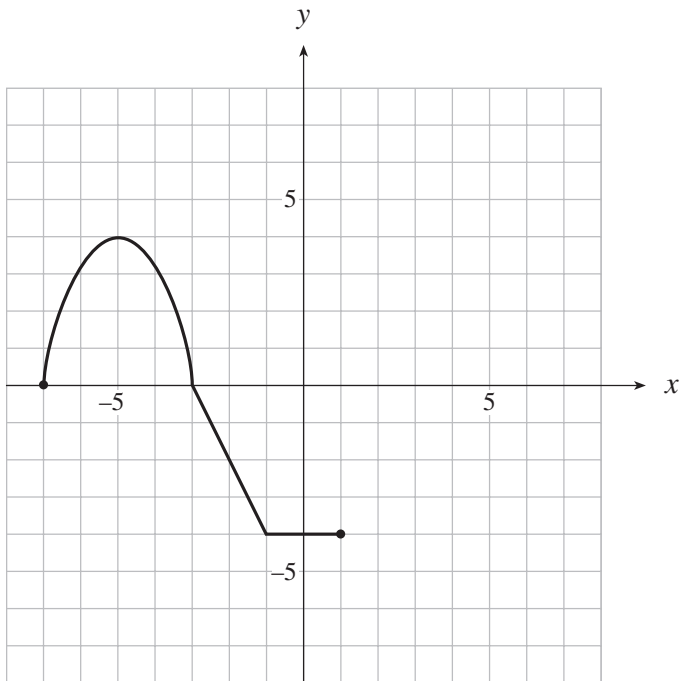
5. The graph of $y = f(x)$ is shown below.



a) Graph: $y = -2f(x + 3)$

(3 marks)

 solution

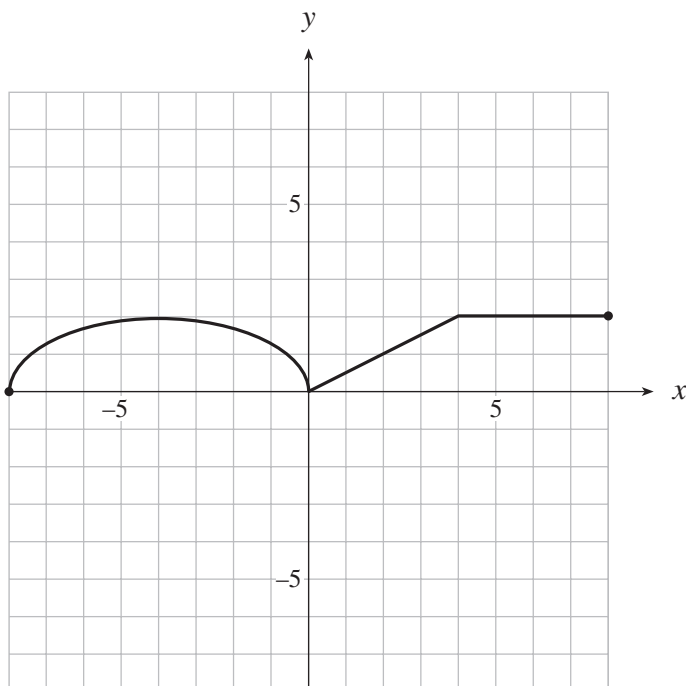


1 mark for the reflection
1 mark for vertical expansion
1 mark for horizontal translation

b) Graph: $y = \left| f\left(\frac{x}{2}\right) \right|$

(2 marks)

 solution



1 mark for horizontal expansion
1 mark for absolute value

6. Solve algebraically, giving exact values, where $0 \leq x < 2\pi$.

(4 marks)

$$2 \tan x \sin x - \tan x = 0$$

 **solution**

$$2 \tan x \sin x - \tan x = 0$$

$$\tan x(2 \sin x - 1) = 0 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\frac{1}{2} \mathbf{mark} \rightarrow \tan x = 0 \quad \sin x = \frac{1}{2} \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

$$x = 0, \pi \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

↑ ↑ ↑ ↑

$\frac{1}{2}$ **mark each** $\frac{1}{2}$ **mark each**

7. Prove the identity:

(4 marks)

$$\frac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$$

 solution

LEFT SIDE

RIGHT SIDE

$$\frac{\cos x + \cot x}{\sec x + \tan x}$$

$$\cos x \cot x$$

$$\underbrace{\left(\cos x + \frac{\cos x}{\sin x} \right)}_{\frac{1}{2} \text{ mark}} \cdot \frac{\sin x \cos x}{\sin x \cos x} \left. \vphantom{\frac{\cos x + \cot x}{\sec x + \tan x}} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$\underbrace{\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right)}_{\frac{1}{2} \text{ mark}}$$

$$\frac{1}{2} \text{ mark} \rightarrow \frac{\sin x \cos^2 x + \cos^2 x}{\sin x + \sin^2 x}$$

$$\frac{1}{2} \text{ mark} \rightarrow \frac{\cos^2 x (\sin x + 1)}{\sin x (1 + \sin x)}$$

$$\frac{1}{2} \text{ mark} \rightarrow \frac{\cos x \cos x}{\sin x}$$

$$\frac{1}{2} \text{ mark} \rightarrow \cos x \cot x$$

LS = RS

7. Prove the identity:

(4 marks)

$$\frac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$$

 **alternate solution**

LEFT SIDE	RIGHT SIDE
$\frac{\cos x + \cot x}{\sec x + \tan x}$	$\cos x \cot x$
$\frac{\cos x + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$ <p style="text-align: center;"> $\frac{1}{2}$ mark \downarrow $\frac{1}{2}$ mark \uparrow </p>	
$\frac{\frac{\cos x \sin x + \cos x}{\sin x}}{\frac{1 + \sin x}{\cos x}} \leftarrow \frac{1}{2} \text{ mark (for combining fractions)}$ <p style="text-align: center;"> $\frac{1}{2}$ mark \downarrow </p>	
$\frac{\cos x \sin x + \cos x}{\sin x} \left(\frac{\cos x}{1 + \sin x} \right)$ <p style="text-align: center;"> $\frac{1}{2}$ mark \downarrow $\frac{1}{2}$ mark for reciprocal \uparrow </p>	
$\left(\frac{\cos x(\sin x + 1)}{\sin x} \right) \left(\frac{\cos x}{1 + \sin x} \right)$	
$\frac{\cos^2 x}{\sin x}$	
$\cos x \cdot \frac{\cos x}{\sin x} \leftarrow \frac{1}{2} \text{ mark}$	
$\cos x \cot x \leftarrow \frac{1}{2} \text{ mark}$	
<p>LS = RS</p>	

END OF KEY