Principles of Mathematics 12

August 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	C	U	1.5	2	C3	23.	B	H	1.5	2	C2
2.	D	K	1.5	2	D6	24.	D	K	1.5	3	E2
3.	A	U	1.5	2	C4	25.	D	H	1.5	3	E2
4.	B	U	1.5	2	D6	26.	C	U	1.5	3	E3
5.	D	U	1.5	2	C8	27.	A	U	1.5	3	E2
6.	B	U	1.5	2	D6	28.	D	K	1.5	3	F1
7.	B	U	1.5	2	C5	29.	D	K	1.5	3	F2
8.	B	H	1.5	2	D7	30.	A	U	1.5	3	F3
9.	D	H	1.5	2	D5	31.	A	U	1.5	3	F6
10.	D	H	1.5	2	C5	32.	D	U	1.5	3	F6
11.	B	U	1.5	2	B1	33.	B	H	1.5	3	F6
12.	B	U	1.5	2	B1	34.	A	H	1.5	4	G5
13.	C	U	1.5	2	B1	35.	B	U	1.5	4	G8
14.	B	U	1.5	2	B3	36.	C	U	1.5	4	G7
15.	A	H	1.5	2	B1, A3	37.	C	U	1.5	4	G11
16.	D	K	1.5	2	D2	38.	C	U	1.5	4	G11
17.	A	U	1.5	2	D3	39.	B	H	1.5	4	G12
18.	A	U	1.5	2	C2	40.	C	U	1.5	4	G9
19.	B	U	1.5	2	C1	41.	C	U	1.5	4	G8
20.	D	U	1.5	2	C2	42.	C	K	1.5	4	G2
21.	D	U	1.5	2	D1	43.	B	U	1.5	4	G2
22.	B	H	1.5	2	D1	44.	C	H	1.5	4	G2

Multiple Choice = 66 marks

Part B: Written Response

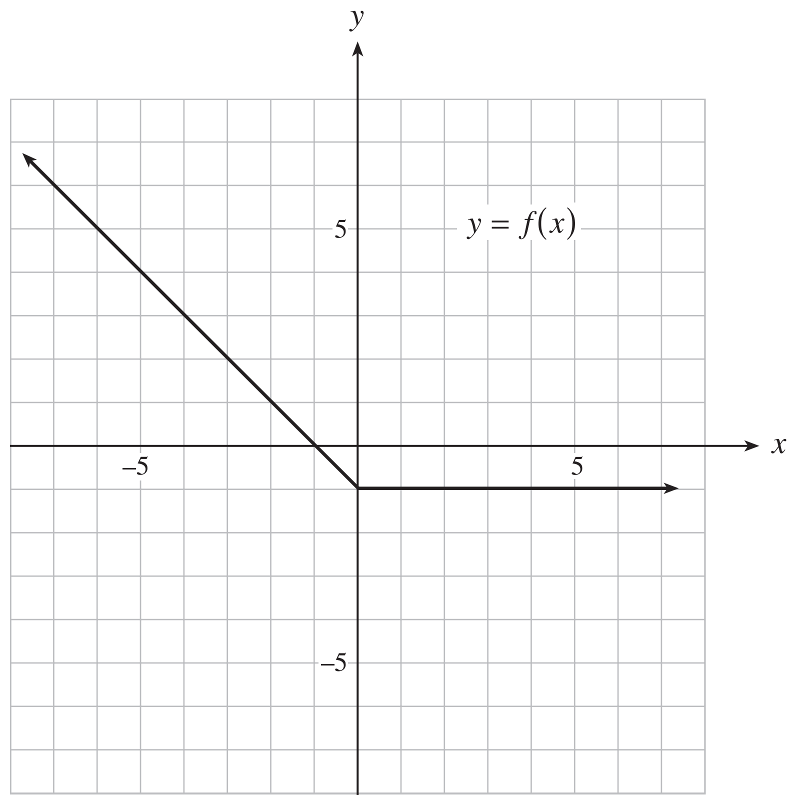
Q	B	C	S	CO	PLO
1a.	1	U	2	3	F4
1b.	2	U	2	3	F2, F5
2.	3	U	4	3	E2
3.	4	U	5	2	D1
4a.	5	U	2	4	G7
4b.	6	U	2	4	G6
5a.	7	U	2	4	G13
5b.	8	U	2	4	G13
6a.	9	U	2	4	G3, G8
6b.	10	U	2	4	G3, G8
7a.	11	U	3	2	C6
7b.	12	U	1	2	C6
8.	13	H	5	2	C7

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

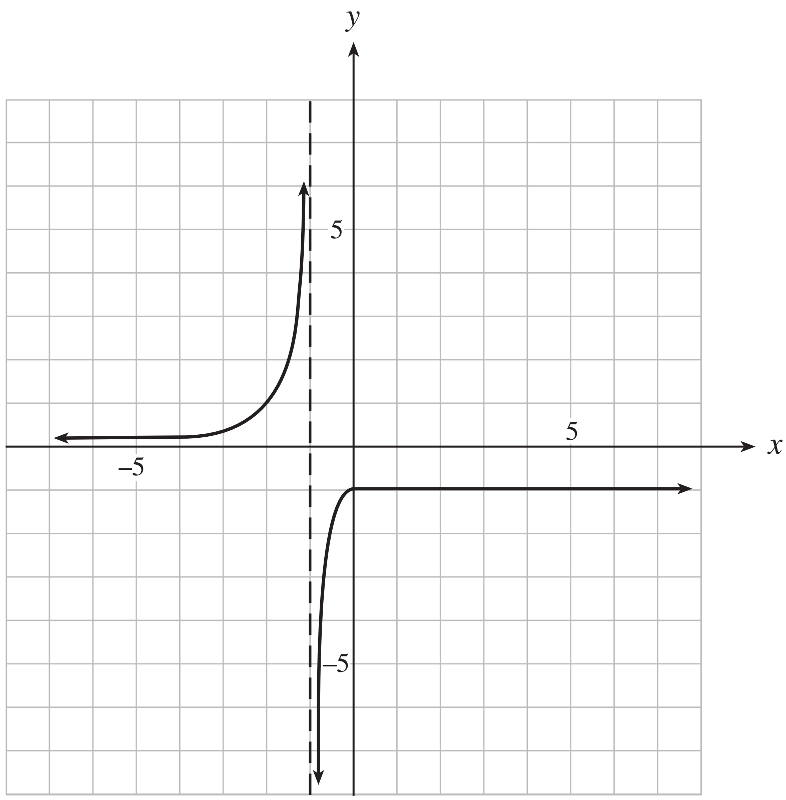
LEGEND:
Q = Question Number | **K** = Keyed Response | **C** = Cognitive Level
B = Score Box Number | **S** = Score | **CO** = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

solution



$\frac{1}{2}$ **mark** for $(-2,\ 1)$ and shape

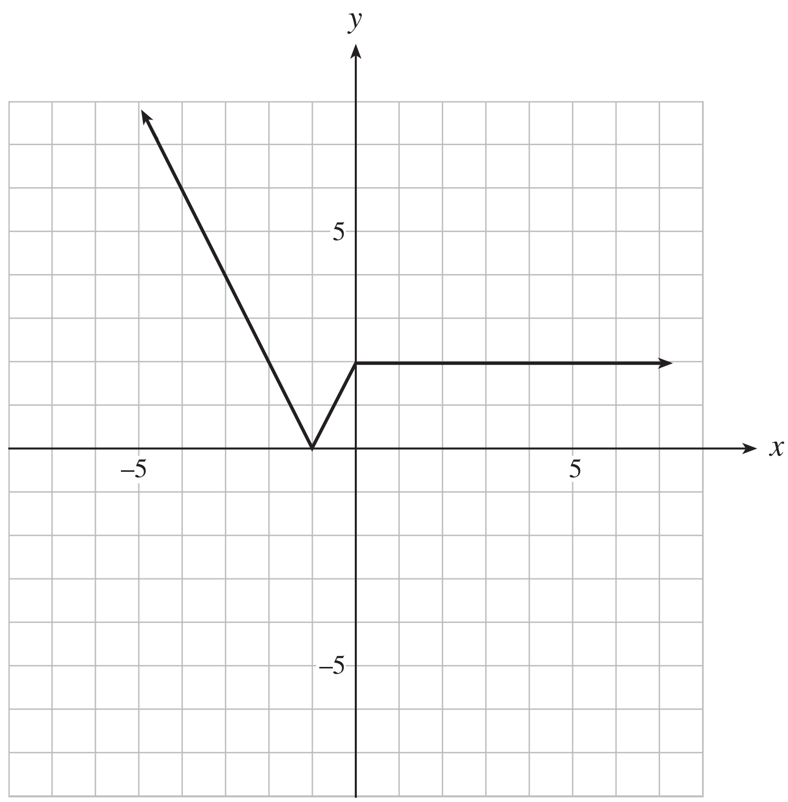
$\frac{1}{2}$ **mark** for $(0,\ -1)$ and shape to left

$\frac{1}{2}$ **mark** for horizontal portion of graph

$\frac{1}{2}$ **mark** for asymptotic behaviour of graph

b) On the grid provided, sketch the graph of $y = 2\left|f(x)\right|$. **(2 marks)**

solution

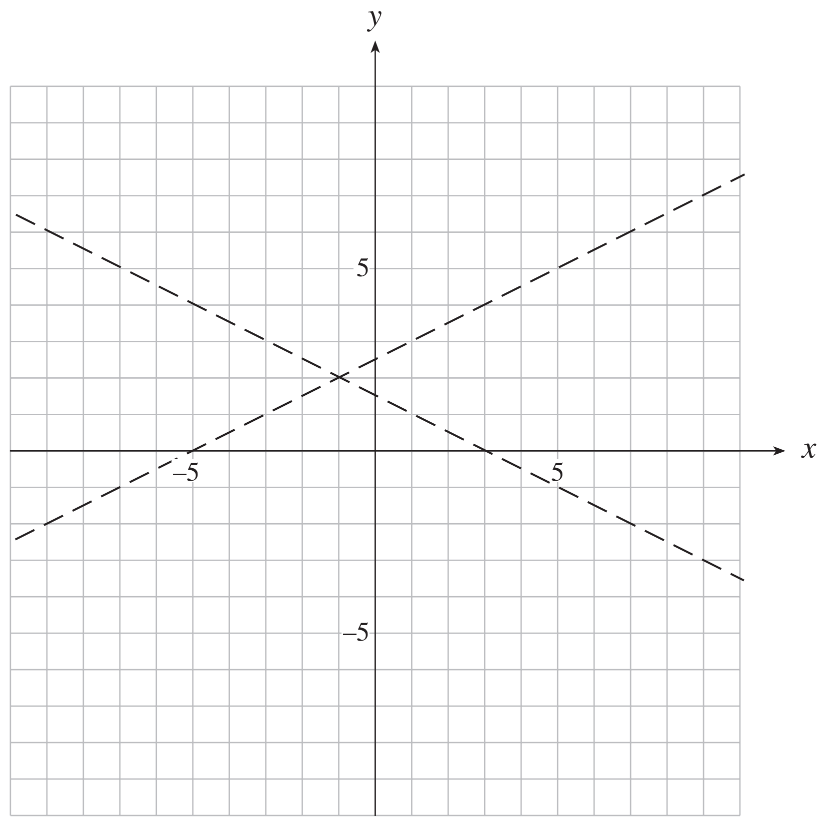


1 mark for absolute value
1 mark for vertical expansion

Note: Arrowheads on solution are not necessary.

2. The asymptotes of a hyperbola are shown below. Determine an equation of the hyperbola if the transverse axis is horizontal and has a length of 8. **(4 marks)**

(**Note:** The grid is provided for rough work only.)



solution

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

↓ ↓

$$\frac{(x+1)^2}{16} - \frac{(y-2)^2}{4} = 1$$ ← **1 mark** for equation format

↑ ↑

1 mark **1 mark**

Note: A detailed graph of the hyperbola is not required.
The equation does not have to be in standard form to receive full marks.

3. Malcolm bought a new car for \$24 000. Every year it will depreciate in value by 8%. How long will it take for the car to be worth \$16 000? **(5 marks)**

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

solution

$$\underset{\uparrow}{\tfrac{1}{2}\textbf{ mark}}\quad \underset{\uparrow}{\textbf{1 mark}}\quad \underset{\uparrow}{\tfrac{1}{2}\textbf{ mark}}$$

$$\overset{\downarrow}{24\,000}\,\overset{\downarrow}{(1-.08)^t} = \overset{\downarrow}{16\,000}$$

$$0.92^t = \frac{16\,000}{24\,000} = \frac{2}{3} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\log 0.92^t = \log\frac{2}{3} \qquad \leftarrow \textbf{1 mark}$$

$$t\log 0.92 = \log\frac{2}{3} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = \frac{\log\frac{2}{3}}{\log 0.92} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = 4.86 \text{ years} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

4. a) How many groups of 3 chairs can be chosen from 7 chairs if the chairs are all different colours? **(2 marks)**

solution

1 mark **1 mark**

↓ **OR** ↓

$$_7C_3 = \frac{7!}{3!\,4!}$$

$= 35$ ← **1 mark**

b) How many different ways can 7 chairs be arranged in a row if 2 of the chairs are blue, 3 are yellow, 1 is red and 1 is green? (Assume that all of the chairs are identical except for colour.) **(2 marks)**

solution

$\frac{1}{2}$ mark

↓

$$\frac{7!}{2!\,3!} = 420 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

↑ ↑

$\frac{1}{2}$ mark each

5. A hand of five cards is dealt from a standard deck of 52 cards.

   a) What is the probability that the hand contains exactly 1 club? **(2 marks)**

solution

$$P(1\text{ club}) = \frac{\overbrace{\left({}_{13}C_1\right)}^{\frac{1}{2}\textbf{ mark}}\overbrace{\left({}_{39}C_4\right)}^{\frac{1}{2}\textbf{ mark}}}{\underbrace{{}_{52}C_5}_{\frac{1}{2}\textbf{ mark}}} \approx 0.4114195678 \approx 0.41 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

b) What is the probability that the hand contains at most 1 club? **(2 marks)**

solution

$$P(\text{at most 1 club}) = P(0\text{ clubs}) + P(1\text{ club})$$

this $\frac{1}{2}$ mark is for *adding* the answer from part a)

$$= \frac{\overbrace{{}_{39}C_5}^{\frac{1}{2}\textbf{ mark}}}{\tfrac{1}{2}\textbf{ mark} \rightarrow {}_{52}C_5} + \overbrace{\frac{\left({}_{13}C_1\right)\left({}_{39}C_4\right)}{{}_{52}C_5}}^{\frac{1}{2}\textbf{ mark}}$$

$$\approx 0.6329531813$$

$$\approx 0.63 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

6. In a large city in BC the probability that a car has air conditioning is 0.72. If 200 cars are randomly selected, determine the probability that between 130 and 132 cars inclusive have air conditioning by using the following methods.

   a) Use the binomial distribution to obtain this probability.
   (Answer accurate to at least 4 decimal places.) **(2 marks)**

solution

$p = 0.72 \qquad n = 200 \quad \leftarrow \tfrac{1}{2}$ **mark**

$P(130 \le X \le 132) = \text{binomcdf}(200,\ 0.72,\ 132) - \text{binomcdf}(200,\ 0.72,\ 129) \quad \leftarrow 1\tfrac{1}{2}$ **marks**

$= 0.0244056415$

≈ 0.0244

OR

$p = 0.72 \qquad n = 200 \quad \leftarrow \tfrac{1}{2}$ **mark**

$P(130 \le X \le 132)$

$= \text{binompdf}(200,\ 0.72,\ 130) + \text{binompdf}(200,\ 0.72,\ 131) + \text{binompdf}(200,\ 0.72,\ 132) \quad \leftarrow 1\tfrac{1}{2}$ **marks**

$= 0.0244056408$

≈ 0.0244

OR

$p = 0.72 \qquad n = 200 \quad \leftarrow \tfrac{1}{2}$ **mark**

$P(x) = {}_nC_x\, p^x q^{n-x} \qquad \Rightarrow$

$P(130 \le X \le 132) = {}_{200}C_{130}(0.72)^{130}(0.28)^{70} + {}_{200}C_{131}(0.72)^{131}(0.28)^{69} + {}_{200}C_{132}(0.72)^{132}(0.28)^{68} \quad \leftarrow 1\tfrac{1}{2}$ **marks**

$= 0.0244056708$

≈ 0.0244

Note: Due to overflow errors, answers were not marked.

b) Use the normal approximation to the binomial distribution to obtain an estimate of this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

solution

$\mu = np = 200(0.72) = 144 \ \leftarrow$ $\frac{1}{2}$ **mark** $\qquad \sigma = \sqrt{npq} = \sqrt{200(0.72)(0.28)} = \sqrt{40.32} \approx 6.3498 \ \leftarrow$ $\frac{1}{2}$ **mark**

$$\begin{aligned} P(130 \le X \le 132) &= \text{normalcdf}\left(129.5,\ 132.5,\ 144,\ \sqrt{40.32}\right) \quad \leftarrow \textbf{1 mark} \\ &= 0.0238648031 \\ &\approx 0.0239 \end{aligned}$$

alternate solution

If z-tables are used to find the normal approximation

$\mu = np = 200(0.72) = 144 \ \leftarrow$ $\frac{1}{2}$ **mark** $\qquad \sigma = \sqrt{npq} = \sqrt{200(0.72)(0.28)} = \sqrt{40.32} \approx 6.3498 \ \leftarrow$ $\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark**

$$z_1 = \frac{129.5 - 144}{\sqrt{40.32}} = -2.283535358 \approx -2.28 \qquad z_2 = \frac{132.5 - 144}{\sqrt{40.32}} = -1.811079767 \approx -1.81$$

$$z_1 = \frac{129.5 - 144}{\sqrt{40.32}} = -2.283535358 \approx -2.28 \qquad z_2 = \frac{132.5 - 144}{\sqrt{40.32}} = -1.811079767 \approx -1.81$$

$$\left.\begin{aligned} P(130 \le X \le 132) &\approx P(-2.28 < Z < -1.81) \\ &= 0.0352 - 0.0113 \\ &= 0.0239 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

7. a) Solve algebraically, giving exact values for x, where $0 \le x < 2\pi$. **(3 marks)**

$$2\cos^2 x - \cos x - 1 = 0$$

solution

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\underset{\tfrac{1}{2}\textbf{ mark}}{\underset{\uparrow}{\cos x = -\frac{1}{2}}} \quad \text{or} \quad \underset{\tfrac{1}{2}\textbf{ mark}}{\underset{\uparrow}{\cos x = 1}}$$

$$x = \underset{\tfrac{1}{2}\textbf{ mark}}{\underset{\uparrow}{\frac{2\pi}{3}}} \quad \text{or} \quad \underset{\tfrac{1}{2}\textbf{ mark}}{\underset{\uparrow}{\frac{4\pi}{3}}} \qquad \underset{\tfrac{1}{2}\textbf{ mark}}{\underset{\uparrow}{x = 0}}$$

b) Give the general solution for this equation.
(Solve over the set of real numbers, giving exact value solutions.) **(1 mark)**

solution

$$\left.\begin{aligned} x &= \frac{2\pi}{3} + 2\pi n \\ x &= \frac{4\pi}{3} + 2\pi n \end{aligned}\right\} \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$x = 2\pi n \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

where $n \in I$

Note: $x = \frac{2\pi n}{3}$ would generate all solutions and receive **1 mark**

It is not necessary for students to state $n \in I$

8. Prove the identity: **(5 marks)**

$$\frac{\cos\theta + \cot\theta}{1+\sin\theta} = \cot\theta$$

solution

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos\theta + \cot\theta}{1+\sin\theta}$	$\cot\theta$

LEFT SIDE:

$$\frac{\cos\theta + \cot\theta}{1+\sin\theta}$$

$\frac{1}{2}$ mark

$$= \frac{\left(\cos\theta + \dfrac{\cos\theta}{\sin\theta}\right)}{(1+\sin\theta)}\ \frac{\sin\theta}{\sin\theta} \quad \leftarrow \textbf{1 mark}$$

$1\frac{1}{2}$ marks $\rightarrow$ $= \dfrac{\cos\theta\sin\theta + \cos\theta}{(1+\sin\theta)\sin\theta}$

1 mark $\rightarrow$ $= \dfrac{\cos\theta(\sin\theta + 1)}{(1+\sin\theta)\sin\theta}$

$\frac{1}{2}$ mark $\rightarrow$ $= \dfrac{\cos\theta}{\sin\theta}$

$\frac{1}{2}$ mark $\rightarrow$ $= \cot\theta$

RIGHT SIDE:

$$\cot\theta$$

LS = RS

8. Prove the identity: **(5 marks)**

$$\frac{\cos\theta+\cot\theta}{1+\sin\theta} = \cot\theta$$

alternate solution 1

LEFT SIDE	RIGHT SIDE
$\frac{\cos\theta+\cot\theta}{1+\sin\theta}$	$\cot\theta$

$\frac{1}{2}$ **mark**

$$= \frac{\cos\theta+\frac{\cos\theta}{\sin\theta}}{1+\sin\theta}\ \frac{(1-\sin\theta)}{(1-\sin\theta)} \leftarrow \textbf{1 mark}$$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos\theta-\sin\theta\cos\theta+\frac{\cos\theta}{\sin\theta}-\cos\theta}{1-\sin^2\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{-\sin\theta\cos\theta+\frac{\cos\theta}{\sin\theta}}{\cos^2\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos\theta\left(-\sin\theta+\frac{1}{\sin\theta}\right)}{\cos^2\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \left(\dfrac{-\sin\theta+\frac{1}{\sin\theta}}{\cos\theta}\right)\dfrac{\sin\theta}{\sin\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{-\sin^2\theta+1}{\cos\theta\sin\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos^2\theta}{\cos\theta\sin\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos\theta}{\sin\theta}$

LS = RS

8. Prove the identity: **(5 marks)**

$$\frac{\cos\theta + \cot\theta}{1+\sin\theta} = \cot\theta$$

alternate solution 2

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos\theta + \cot\theta}{1+\sin\theta}$	$\cot\theta$

3 marks →

$$= \frac{\cos\theta + \dfrac{\cos\theta}{\sin\theta}}{1+\sin\theta}$$

$$= \frac{\dfrac{\sin\theta\cos\theta + \cos\theta}{\sin\theta}}{1+\sin\theta}$$

1 mark → $= \dfrac{\cos\theta(1+\sin\theta)}{\sin\theta} \cdot \dfrac{1}{1+\sin\theta}$

$\frac{1}{2}$ mark → $= \dfrac{\cos\theta}{\sin\theta}$

$\frac{1}{2}$ mark → $= \cot\theta$

LS = RS

8. Prove the identity: **(5 marks)**

$$\frac{\cos\theta + \cot\theta}{1+\sin\theta} = \cot\theta$$

alternate solution 3

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos\theta + \cot\theta}{1+\sin\theta}$	$\cot\theta$
3 marks → $= \dfrac{\dfrac{\sin\theta}{\tan\theta} + \dfrac{1}{\tan\theta}}{1+\sin\theta}$	
1 mark → $= \dfrac{\sin\theta + 1}{\tan\theta} \cdot \dfrac{1}{1+\sin\theta}$	
$\frac{1}{2}$ mark → $= \dfrac{1}{\tan\theta}$	
$\frac{1}{2}$ mark → $= \cot\theta$	

LS = RS

END OF KEY