

Principles of Mathematics 12

June 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	2	C3	23.	B	H	1.5	2	D3
2.	B	K	1.5	2	D6	24.	C	K	1.5	3	E1
3.	A	U	1.5	2	C3	25.	B	K	1.5	3	E2
4.	B	U	1.5	2	C5, A9	26.	D	U	1.5	3	E2
5.	C	U	1.5	2	C5	27.	D	H	1.5	3	E2
6.	D	U	1.5	2	D7	28.	B	K	1.5	3	F3
7.	C	U	1.5	2	C7	29.	A	U	1.5	3	F5
8.	D	U	1.5	2	D5	30.	A	U	1.5	3	F6
9.	D	H	1.5	2	C8	31.	B	U	1.5	3	F2, F1
10.	D	H	1.5	2	D6	32.	B	H	1.5	3	F1, F4
11.	A	U	1.5	2	B1	33.	B	H	1.5	3	F2, A2
12.	A	U	1.5	2	B1	34.	C	K	1.5	4	G8
13.	C	U	1.5	2	B1	35.	C	U	1.5	4	G6
14.	C	H	1.5	2	B1, D4	36.	B	U	1.5	4	G4
15.	C	U	1.5	2	B3	37.	A	K	1.5	4	G11
16.	D	U	1.5	2	D4	38.	C	U	1.5	4	G11
17.	A	K	1.5	2	C2	39.	B	U	1.5	4	G11
18.	D	K	1.5	2	D3	40.	C	H	1.5	4	G13
19.	D	U	1.5	2	D4	41.	B	K	1.5	4	G2
20.	D	U	1.5	2	C1	42.	A	U	1.5	4	G1
21.	C	U	1.5	2	D3	43.	C	U	1.5	4	G2
22.	D	U	1.5	2	D1	44.	A	H	1.5	4	G2

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F6
1b.	2	U	2	3	F3
2.	3	U	4	3	E3
3.	4	U	5	2	D1
4a.	5	U	2	4	G7
4b.	6	U	2	4	G7
5a.	7	U	2	4	G12
5b.	8	U	2	4	G12
6a.	9	U	2	4	G3, G8
6b.	10	U	2	4	G3, G8
7a.	11	U	3	2	C5
7b.	12	U	1	2	C6
8.	13	H	5	2	C7

Written Response = 34 marks

Multiple Choice = 66 (44 questions)

Written Response = 34 (8 questions)

EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

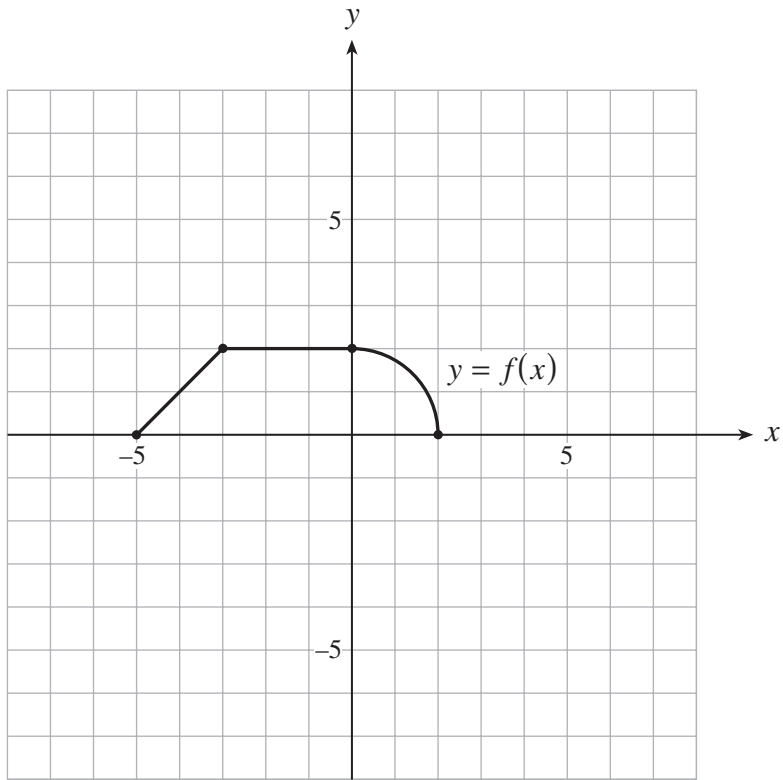
K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

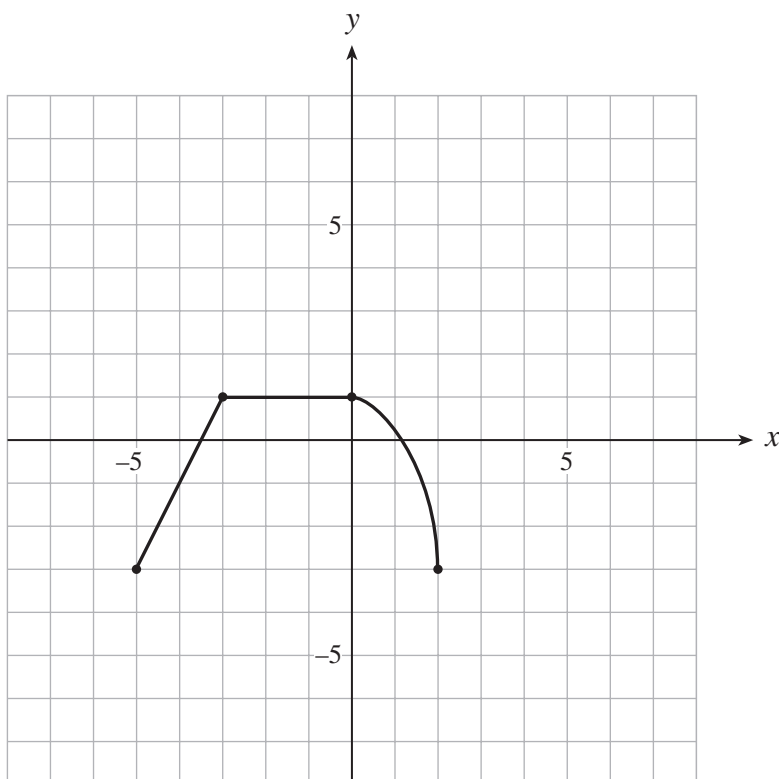
1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = 2f(x) - 3$.

(2 marks)

 solution

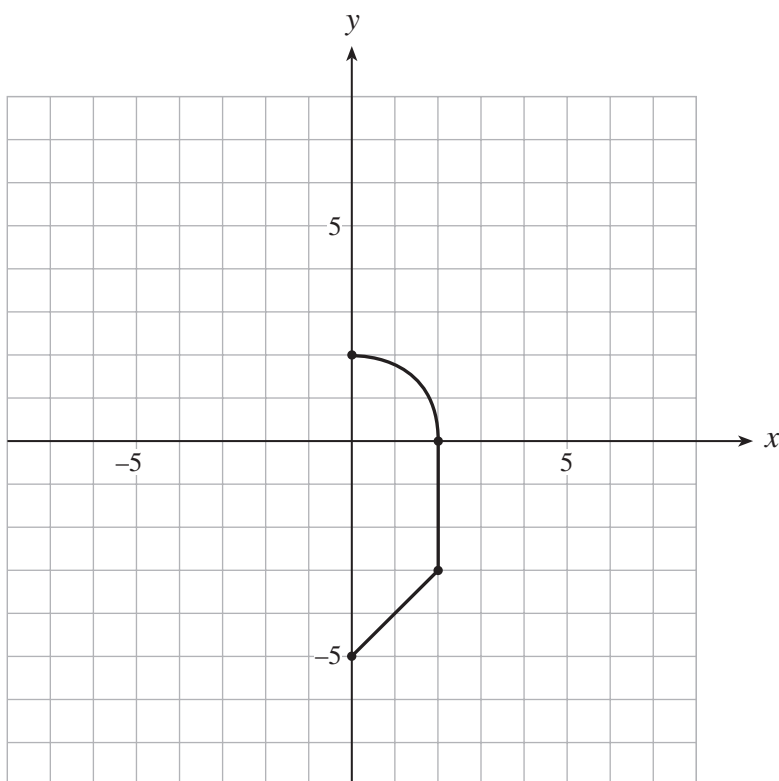


1 mark for vertical expansion
1 mark for vertical translation

b) On the grid provided, sketch the inverse relation of $y = f(x)$.

(2 marks)

70 solution



1 mark shape/orientation
1 mark location

2. Change to standard form:

(4 marks)

$$3x^2 - 2y^2 - 6x - 12y - 27 = 0$$

 solution

$$3x^2 - 2y^2 - 6x - 12y - 27 = 0$$

$$3(x^2 - 2x \quad) - 2(y^2 + 6y \quad) = 27$$

↑
 $\frac{1}{2}$ mark

↑
 $\frac{1}{2}$ mark

$$3(x^2 - 2x + 1) - 2(y^2 + 6y + 9) = 27 + 3 - 18$$

↑
 $\frac{1}{2}$ mark

↑
 $\frac{1}{2}$ mark

↑ ↑
 $\frac{1}{2}$ mark each

$$3(x-1)^2 - 2(y+3)^2 = 12 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\frac{(x-1)^2}{4} - \frac{(y+3)^2}{6} = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

3. If 200 g of a substance decays to 17 g in 28 days, determine the half-life of this substance.
(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) **(5 marks)**

 **solution**

$$\begin{array}{c} \frac{1}{2} \text{ mark} \\ \downarrow \\ 17 = 200 \left(\frac{1}{2} \right)^{\frac{28}{n}} \leftarrow \mathbf{1 \text{ mark}} \\ \uparrow \\ \frac{1}{2} \text{ mark} \end{array}$$

$$\frac{1}{2} \text{ mark} \rightarrow \frac{17}{200} = \left(\frac{1}{2} \right)^{\frac{28}{n}}$$

$$\log \frac{17}{200} = \frac{28}{n} \log \frac{1}{2} \quad \leftarrow \mathbf{1 \frac{1}{2} \text{ marks}}$$

$$\left. \begin{array}{l} n = \frac{28 \log \frac{1}{2}}{\log \frac{17}{200}} \\ n = 7.87 \text{ days} \end{array} \right\} \leftarrow \mathbf{1 \text{ mark}}$$

4. There are 7 boys and 5 girls in a group of students.

- a) Calculate the number of ways that a committee of 4 students can be chosen from this group if the committee must have exactly 1 boy. **(2 marks)**

 **solution**

$\frac{1}{2}$ mark each

↓ ↓

$$({}_7C_1)({}_5C_3) = 70$$

↑ ↑
 $\frac{1}{2}$ mark $\frac{1}{2}$ mark

 **alternate solution**

$\frac{1}{2}$ mark each

↓ ↓

$$7 \times \frac{5 \times 4 \times 3}{3!} = 70$$

↑ ↑
 $\frac{1}{2}$ mark $\frac{1}{2}$ mark

- b) If the committee of 4 students must have a female president, a male vice-president, and 2 other members chosen from the remaining students, how many ways can such a committee be chosen? **(2 marks)**

 **solution**

$\frac{1}{2}$ mark

↓ 1 mark

$$({}_7C_1)({}_5C_1)({}_{10}C_2) = 1575$$

↑
 $\frac{1}{2}$ mark

 **alternate solution**

$\frac{1}{2}$ mark

$$7 \times 5 \times \frac{10 \times 9}{2!} = 1575$$

↑ ↑
1 mark $\frac{1}{2}$ mark

Note: ${}_7C_1$ could be written as ${}_7P_1$

5. Bag A contains 5 white balls and 2 green balls. Bag B contains 3 white balls and 4 green balls. A fair die is rolled and if a 1 or 2 comes up, a ball is randomly selected from Bag A; however, if a 3, 4, 5 or 6 comes up, a ball is randomly selected from Bag B.



Bag A

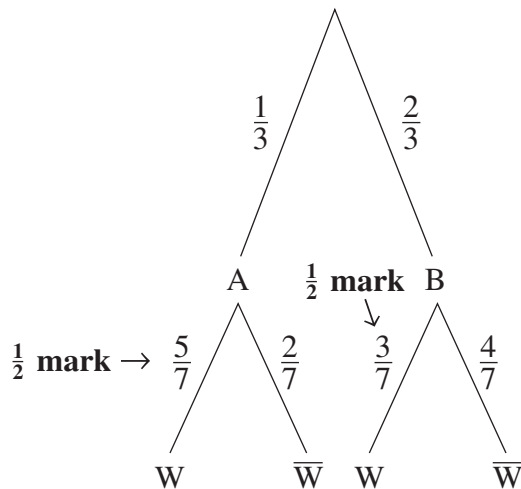


Bag B

a) What is the probability of selecting a white ball?

(2 marks)

no solution



$$P(W) = P(A \text{ and } W) + P(B \text{ and } W)$$

$$= \frac{1}{3} \times \frac{5}{7} + \frac{2}{3} \times \frac{3}{7} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= \frac{11}{21} \quad \leftarrow \frac{1}{2} \text{ mark}$$

or

$$0.52$$

b) If a white ball is selected, what is the probability that this ball came from Bag A? **(2 marks)**

 **solution**

$$P(A|W) = \frac{P(A \text{ and } W)}{P(W)} \quad \leftarrow \frac{1}{2} \text{ mark for recognition of conditional probability}$$

$$= \frac{\frac{1}{3} \times \frac{5}{7}}{\frac{11}{21}} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= \frac{5}{11} \quad \leftarrow \frac{1}{2} \text{ mark}$$

or

0.45

6. Sixty-four percent (64%) of Principles of Mathematics students also take a Calculus course. If 175 Principles of Mathematics students are randomly selected, determine the probability that exactly 110 of these students also take a Calculus course by using the following methods.

- a) Use the binomial distribution to obtain this probability.
(Answer accurate to at least 4 decimal places.)

(2 marks)

solution

$$p = 0.64 \quad n = 175 \quad \leftarrow \frac{1}{2} \text{ mark each}$$

$$\begin{aligned} P(\text{exactly 110 take Calculus}) &= \text{binompdf}(175, 0.64, 110) \quad \leftarrow \mathbf{1 \text{ mark}} \left(\begin{array}{l} \frac{1}{2} \text{ mark for form} \\ \frac{1}{2} \text{ mark for 110} \end{array} \right) \\ &= 0.0593132528 \\ &\approx 0.0593 \end{aligned}$$

alternate solution

$$p = 0.64 \quad n = 175 \quad \leftarrow \frac{1}{2} \text{ mark each}$$

$$\begin{aligned} P(x) = {}_n C_x p^x q^{n-x} \quad \Rightarrow \quad P(110) &= {}_{175} C_{110} (0.64)^{110} (0.36)^{65} \quad \leftarrow \mathbf{1 \text{ mark}} \left(\begin{array}{l} \frac{1}{2} \text{ mark for form} \\ \frac{1}{2} \text{ mark for 110, 65} \end{array} \right) \\ &= 0.0593132527 \\ &\approx 0.0593 \end{aligned}$$

Note: Overflow problems were encountered on some calculators.
Full marks were given for expression shown.

b) Use the normal approximation to the binomial to obtain an estimate of this probability.
(Answer accurate to at least 4 decimal places.) **(2 marks)**

solution

$$\mu = np = 175(0.64) = 112 \quad \sigma = \sqrt{npq} = \sqrt{175(0.64)(0.36)} = 6.349803147 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$P(\text{exactly 110 take Calculus}) \approx \text{normalcdf}(109.5, 110.5, 112, 6.349803147) \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$= 0.0597314797$$

$$\approx 0.0597$$

$\left(\begin{array}{l} \frac{1}{2} \text{ mark form} \\ \frac{1}{2} \text{ mark continuity correction} \end{array} \right)$

alternate solution

If z -tables are used to find the normal approximation:

$$\mu = np = 175(0.64) = 112 \quad \sigma = \sqrt{npq} = \sqrt{175(0.64)(0.36)} = 6.349803147 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$z_1 = \frac{109.5 - 112}{6.349803147} \approx -0.3937 \approx -0.39 \quad z_2 = \frac{110.5 - 112}{6.349803147} \approx -0.2362 \approx -0.24$$

$$P(\text{exactly 110 take Calculus}) \approx P(-0.39 < Z < -0.24) \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$= 0.4052 - 0.3483$$

$$= 0.0569$$

7. a) Solve algebraically, giving exact values for x , where $0 \leq x < 2\pi$.

(3 marks)

$$2 \sin^2 x - \sin x = 0$$

 **solution**

$$2 \sin^2 x - \sin x = 0$$

$$\sin x(2 \sin x - 1) = 0$$

$$\frac{1}{2} \text{ mark} \rightarrow \sin x = 0 \quad \sin x = \frac{1}{2} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = 0, \pi \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\underbrace{\hspace{15em}}_{\uparrow} \\ \frac{1}{2} \text{ mark for each solution}$$

b) Give the general solution for this equation.

(Solve over the set of real numbers, giving exact value solutions.)

(1 mark)

 **solution**

$$x = n\pi \quad x = \frac{\pi}{6} + 2n\pi, \quad x = \frac{5\pi}{6} + 2n\pi, \quad \text{where } n \in I$$

$$\underbrace{\hspace{15em}}_{\uparrow} \quad \underbrace{\hspace{15em}}_{\uparrow}$$

$\frac{1}{2}$ mark

$\frac{1}{2}$ mark

or

$$x = 0 + 2n\pi$$

$$x = \pi + 2n\pi$$

Note: it is not necessary for students to state $n \in I$

8. Prove:

(5 marks)

$$\frac{\cot \theta}{\sin \theta - \csc \theta} = -\sec \theta$$

 solution

LEFT SIDE	RIGHT SIDE
$\frac{\cot \theta}{\sin \theta - \csc \theta}$	$-\sec \theta$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta}{\sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1}{\sin \theta - \frac{1}{\sin \theta}}$	
$1 \text{ mark} \rightarrow = \frac{\left(\frac{\cos \theta}{\sin \theta}\right)(\sin \theta)}{\left(\sin \theta - \frac{1}{\sin \theta}\right)(\sin \theta)}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta}{\sin^2 \theta - 1} \leftarrow \frac{1}{2} \text{ mark}$	
$1 \text{ mark} \rightarrow = \frac{\cos \theta}{-\cos^2 \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{-1}{\cos \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = -\sec \theta$	
LS = RS	

END OF KEY