

Principles of Mathematics 12

April 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C4	23.	B	H	1.5	2	D3, C6, A2
2.	D	U	1.5	2	C8	24.	C	K	1.5	3	E2
3.	C	U	1.5	2	C3	25.	D	U	1.5	3	E2
4.	A	U	1.5	2	D6	26.	D	U	1.5	3	E3
5.	D	U	1.5	2	C5, A9	27.	D	H	1.5	3	E2
6.	D	U	1.5	2	C8	28.	B	K	1.5	3	F3
7.	D	U	1.5	2	C5	29.	A	U	1.5	3	F3
8.	C	U	1.5	2	D5	30.	A	U	1.5	3	F2
9.	B	H	1.5	2	D6	31.	A	U	1.5	3	F4
10.	B	H	1.5	2	C4, C6	32.	C	U	1.5	3	F5
11.	C	U	1.5	2	B1	33.	A	H	1.5	3	F6
12.	C	U	1.5	2	B1	34.	B	K	1.5	4	G7
13.	C	U	1.5	2	B1	35.	B	U	1.5	4	G5
14.	D	U	1.5	2	B1	36.	B	U	1.5	4	G5
15.	A	H	1.5	2	B3	37.	A	U	1.5	4	G11
16.	C	K	1.5	2	C2	38.	D	K	1.5	4	G8
17.	B	K	1.5	2	D3	39.	C	U	1.5	4	G13
18.	C	U	1.5	2	C2	40.	A	H	1.5	4	G12
19.	B	U	1.5	2	D1	41.	B	K	1.5	4	G8
20.	A	H	1.5	2	C1	42.	A	U	1.5	4	G2
21.	D	U	1.5	2	D1	43.	D	U	1.5	4	G1
22.	B	U	1.5	2	D4	44.	C	H	1.5	4	G3

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F1
1b.	2	U	2	3	F2, F3
2.	3	U	4	3	E3
3.	4	U	5	2	C2
4.	5	U	4	4	G8
5.	6	U	4	4	G12
6a.	7	U	2	4	G2
6b.	8	U	2	4	G2
7a.	9	U	3	2	D7
7b.	10	U	1	2	D7
8.	11	H	5	2	C7

Written Response = 34 marks

Multiple Choice = 66 (44 questions)

Written Response = 34 (8 questions)

EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

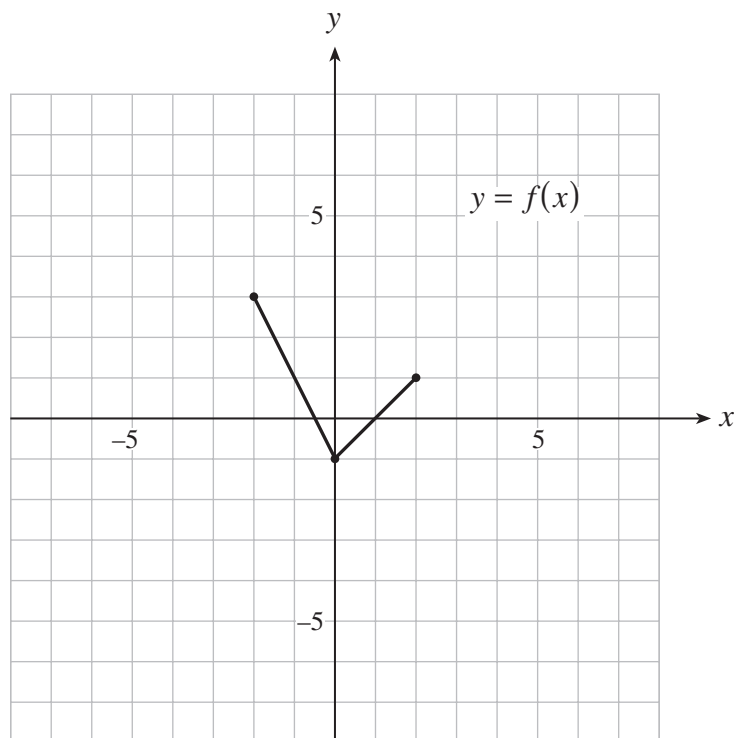
K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

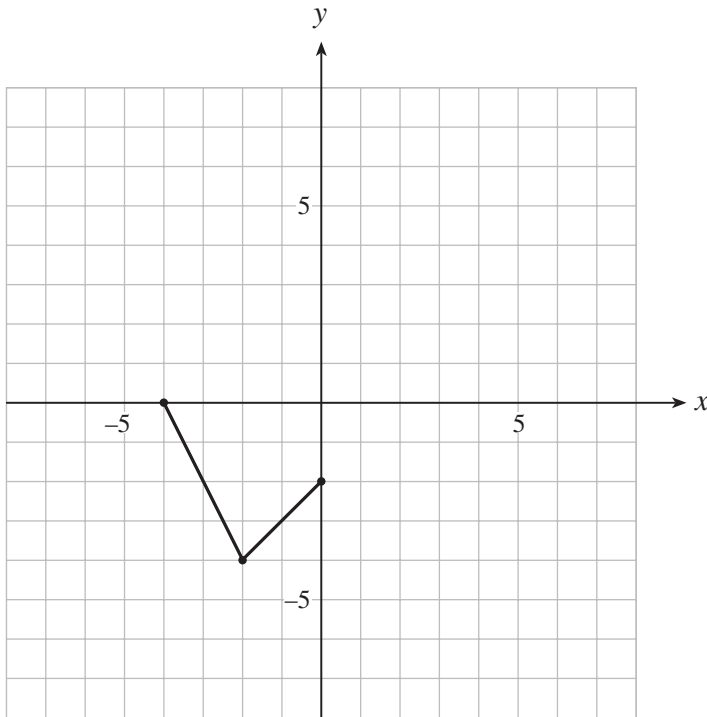
1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = f(x + 2) - 3$.

(2 marks)

 solution



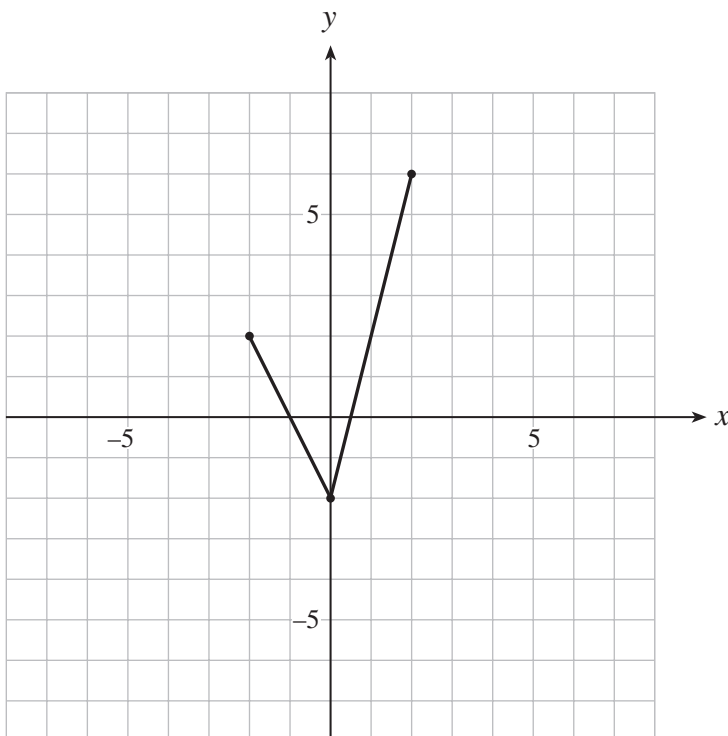
1 mark for horizontal translation

1 mark for vertical translation

b) On the grid provided, sketch the graph of $y = 2f(-x)$.

(2 marks)

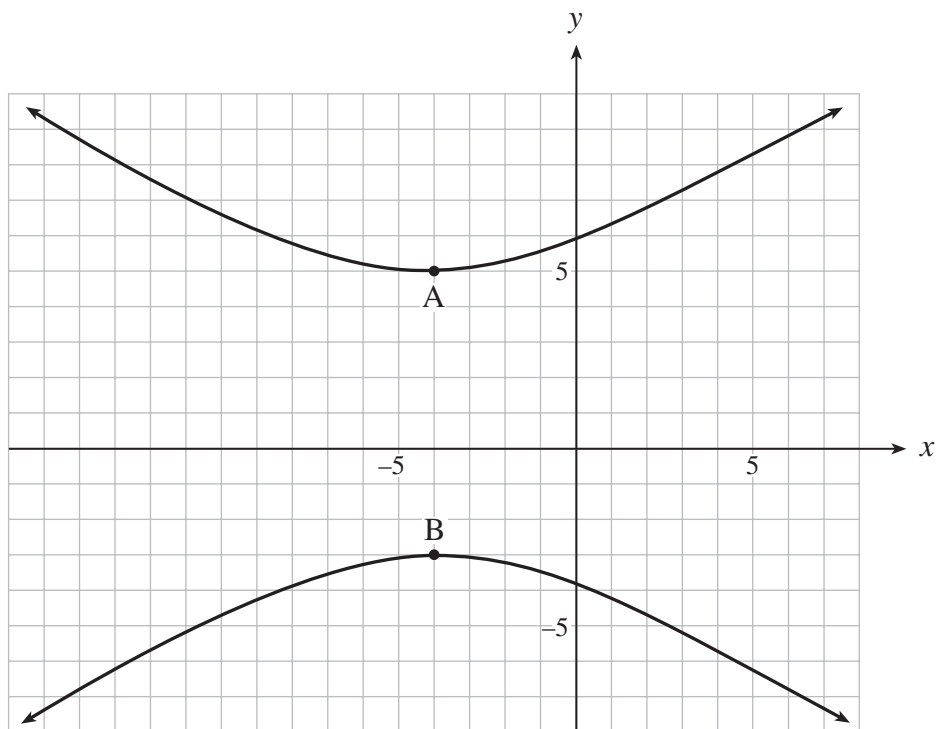
 solution

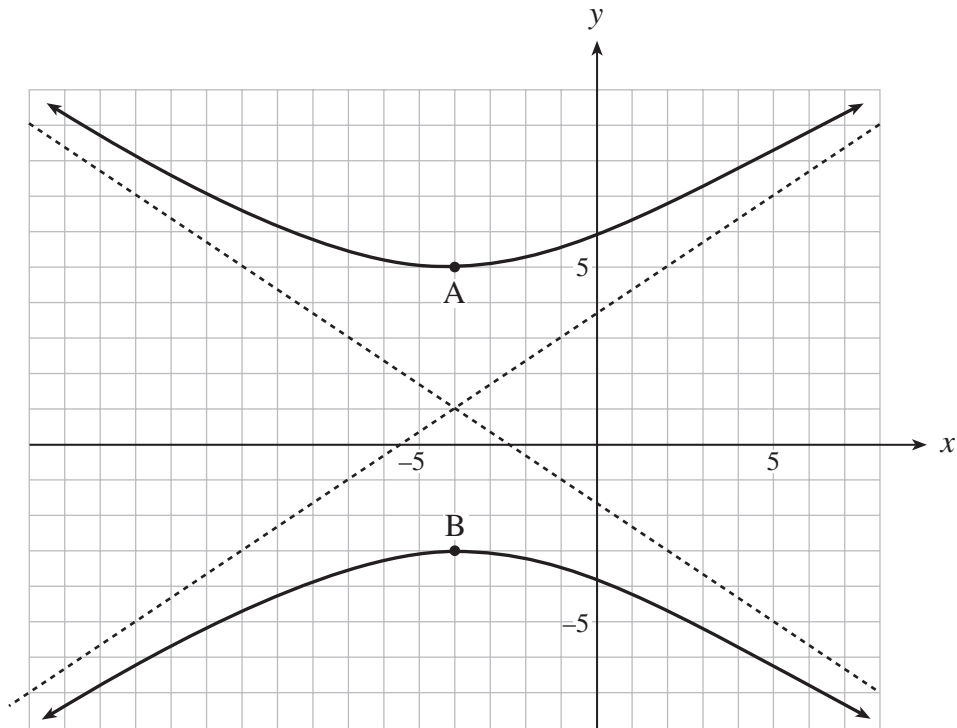


1 mark for reflection in y-axis

1 mark for vertical expansion

2. Determine an equation of the hyperbola graphed below if the slopes of the asymptotes are $\pm \frac{2}{3}$ and the vertices are points A and B. (4 marks)





$\frac{1}{2}$ mark

↓

$$\frac{(x+4)^2}{36}$$

↑
1 mark

$\frac{1}{2}$ mark

↓

$$\frac{(y-1)^2}{16}$$

↑
1 mark

$$= -1 \quad \leftarrow \text{1 mark for equation format}$$

3. Solve algebraically: $\log_2 x = 3 - \log_2(x + 2)$

(5 marks)

solution

$$\log_2 x = 3 - \log_2(x + 2)$$

$$\log_2 x + \log_2(x + 2) = 3 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\log_2(x^2 + 2x) = 3 \quad \leftarrow 1 \text{ mark}$$

$$x^2 + 2x = 8 \quad \leftarrow 1 \text{ mark}$$

$$x^2 + 2x - 8 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(x + 4)(x - 2) = 0$$

$$\begin{array}{l} \frac{1}{2} \text{ mark} \rightarrow x = -4, \quad x = 2 \\ \quad \downarrow \quad \quad \uparrow \\ \quad \emptyset \quad \quad \frac{1}{2} \text{ mark} \\ \quad \text{reject} \\ \quad \uparrow \\ \quad 1 \text{ mark} \end{array}$$

$$\therefore x = 2$$

alternate solution

$$\log_2 x = 3 - \log_2(x + 2)$$

$$\log_2 x = \log_2 8 - \log_2(x + 2) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\log_2 x = \log_2 \frac{8}{x + 2} \quad \leftarrow 1 \text{ mark}$$

$$x = \frac{8}{x + 2} \quad \leftarrow 1 \text{ mark}$$

$$x^2 + 2x = 8$$

$$x^2 + 2x - 8 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(x + 4)(x - 2) = 0$$

$$\begin{array}{l} \frac{1}{2} \text{ mark} \rightarrow x = -4, \quad x = 2 \\ \quad \downarrow \quad \quad \uparrow \\ \quad \emptyset \quad \quad \frac{1}{2} \text{ mark} \\ \quad \text{reject} \\ \quad \uparrow \\ \quad 1 \text{ mark} \end{array}$$

$$\therefore x = 2$$

4. Determine the first 3 terms of the expansion: $(x - 2y)^7$

(4 marks)

 solution

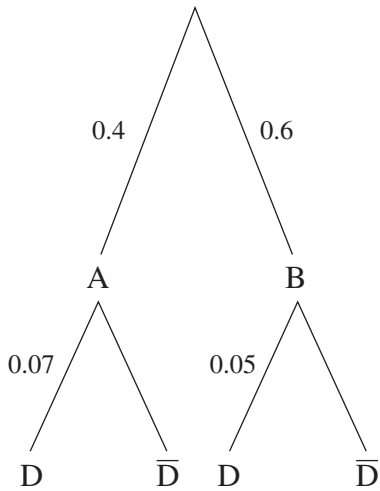
$$\begin{aligned}(x - 2y)^7 &= {}_7C_0 x^7(-2y)^0 + {}_7C_1 x^6(-2y)^1 + {}_7C_2 x^5(-2y)^2 + \dots \\ &= x^7 + 7x^6(-2y) + 21x^5(4y^2) + \dots \\ &= x^7 - 14x^6y + 84x^5y^2 + \dots\end{aligned}$$

$\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} \\ \frac{1}{2} \text{ mark} & & \frac{1}{2} \text{ mark} & & \frac{1}{2} \text{ mark} & & \frac{1}{2} \text{ mark} \\ & & \frac{1}{2} \text{ mark} & & 1 \text{ mark} & & \end{array}$

Note to markers: it is not necessary for students to write + ...

5. A building supply store buys 40% of its pine boards from sawmill A and 60% from sawmill B. Due to pine beetle infestation, 7% of the boards from sawmill A and 5% from sawmill B have a blue discoloration. If a randomly picked board is discoloured, what is the probability that it came from sawmill A? **(4 marks)**

 **solution**



$$P(\text{discoloured from A}) = (0.4)(0.07) = 0.028 \quad \leftarrow \text{1 mark}$$

$$P(\text{discoloured from B}) = (0.6)(0.05) = 0.030 \quad \leftarrow \text{1 mark}$$

$$P(\text{discoloured}) = (0.4)(0.07) + (0.6)(0.05)$$

$$= 0.028 + 0.030$$

$$= 0.058$$

$\leftarrow \text{1 mark}$

$$P(A|D) = \frac{0.028}{0.058}$$

$\leftarrow \text{1 mark}$

$$= 0.4827586207$$

$$\approx 0.48$$

7. A Ferris wheel has a radius of 25 m and its centre is 27 m above the ground. It rotates once every 40 seconds. Sandy gets on the Ferris wheel at its lowest point and then the wheel starts to rotate.
- a) Determine a sinusoidal equation that gives Sandy's height, h , above the ground as a function of the elapsed time, t , where h is in metres and t is in seconds. **(3 marks)**

 **solution**

$$\begin{array}{ccc}
 \frac{1}{2} \text{ mark} & \mathbf{1 \text{ mark}} & \frac{1}{2} \text{ mark} \\
 \downarrow & \downarrow & \downarrow \\
 h = -25 \cos \frac{2\pi}{40} t + 27 \\
 \uparrow & & \uparrow \\
 & \mathbf{1 \text{ mark}} &
 \end{array}$$

- b) Determine the first time, t (in seconds), when Sandy will be 35 m above the ground. **(1 mark)**

 **solution**

$$\begin{array}{l}
 35 = -25 \cos \frac{2\pi}{40} t + 27 \quad \leftarrow \frac{1}{2} \text{ mark} \\
 t = 12.07 \text{ s} \quad \leftarrow \frac{1}{2} \text{ mark}
 \end{array}$$

Note to markers: answer only is acceptable for full marks. No deduction for units missing.

8. Prove the identity:

(5 marks)

$$\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} = 2 \tan^2 x$$

 solution

LEFT SIDE	RIGHT SIDE
$\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x}$	$2 \tan^2 x$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\sin x(1 + \sin x) - \sin x(1 - \sin x)}{1 - \sin^2 x}$	
$1 \text{ mark} \rightarrow = \frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{\cos^2 x}$	
$1 \text{ mark} \rightarrow = \frac{2 \sin^2 x}{\cos^2 x}$	
$\frac{1}{2} \text{ mark} \rightarrow = 2 \tan^2 x$	
$1 \text{ mark} \rightarrow = 2 \tan^2 x$	
LS = RS	

END OF KEY